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REVIEW

In practice

Textbook, p. 2 to 19

Review 1
Waves

Textbook, p. 4

1. *a)* True

   b) False

   c) True

   d) False. Mechanical waves cannot.

Textbook, p. 5

2. Given that from A to C, there are two cycles, two complete cycles will have been covered at point C (according to the legend in Figure 3). Therefore

$$f = \frac{2 \text{ cycles}}{10 \text{ s}} = 0.2 \text{ cycle/s} = 0.2 \text{ Hz}.$$

Textbook, p. 7

3. $|q| = 5 \times 10^{-7}$ C $\quad r = 10$ cm $= 0.10$ m
   $k = 9 \times 10^{9}$ Nm²/C² $\quad E = ?$

$$E = \frac{k|q|}{r^2} = \frac{9 \times 10^{9} \frac{\text{N} \times \text{m}^2}{\text{C}^2} \times 5 \times 10^{-7} \text{ C}}{(0.10 \text{ m})^2}$$

$$= 4.5 \times 10^{5} \text{ N/C}$$

Review 2
Properties of light

Textbook, p. 9

4. If the angle between an incident ray and the surface of a plane mirror is 30°, this means that the angle of incidence is $\theta_i = 90° - 30° = 60°$.

   According to the second law of reflection, the angle of reflection is equal to the angle of incidence. Therefore, $\theta_r = \theta_i = 60°$. The angle of reflection is equal to 60°.

Review 3
Force and motion

Textbook, p. 12

5. $v = 8$ m/s $\quad d = 60$ m $\quad \Delta t = ?$

$$v = \frac{d}{\Delta t} \Rightarrow \Delta t = \frac{d}{v} = \frac{60 \text{ m}}{8 \text{ m/s}} = 7.5 \text{ s}$$

6. *a)* $d = 50$ m $\quad \Delta t = 13$ s $\quad v_{ave} = ?$

$$v_{ave} = \frac{d}{\Delta t} = \frac{50 \text{ m}}{13 \text{ s}} = 3.8 \text{ m/s}$$

   b) $d = 255$ m $\quad \Delta t = 90$ s $\quad v_{ave} = ?$

$$v_{ave} = \frac{d}{\Delta t} = \frac{255 \text{ m}}{90 \text{ s}} = 2.8 \text{ m/s}$$

   c) $d = 300$ km $\quad \Delta t = 2$ h 55 min $= 2.92$ h $\quad v_{ave} = ?$

$$v_{ave} = \frac{d}{\Delta t} = \frac{300 \text{ km}}{2.92 \text{ h}} = 103 \text{ km/h}$$

7. $d = 210$ km $\quad \Delta t = 2$ h 15 min $= 2.25$ h $\quad v_{ave} = ?$

   a) $v_{ave} = \frac{d}{\Delta t} = \frac{210 \text{ km}}{2.25 \text{ h}} = 93.3 \text{ km/h}$

   b) $v_{ave} = 93.3 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 25.9 \text{ m/s}$

Textbook, p. 13

8. $m = 410$ kg $\quad F_g = ?$

   $F_g = mg = 410 \text{ kg} \times 9.80 \text{ m/s}^2 = 4.02 \times 10^{3} \text{ N}$

Textbook, p. 15

9. $F = 50$ N $\quad \theta = 37° \quad F_{eff} = ?$

   $F_{eff} = F\cos\theta = 50 \text{ N} \times \cos 37° = 40 \text{ N}$

10. $F = 20$ N $\quad \theta = 22° \quad F_{eff} = ?$

   $F_{eff} = F\cos\theta = 20 \text{ N} \times \cos 22° = 19 \text{ N}$

Review 4
Energy

Textbook, p. 16

11. $F = 20\text{ N}$ $d = 4\text{ m}$ $W = ?$

$W = Fd = 20\text{ N} \times 4\text{ m} = 80\text{ J}$

12. $F = 100\text{ N}$ $d = 5\text{ m}$ $\Delta E = ?$

$W = Fd = 100\text{ N} \times 5\text{ m} = 500\text{ J}$

$\Delta E = W = 500\text{ J}$

Textbook, p. 17

13. $m = 0.5\text{ kg}$ $v = 2\text{ m/s}$ $E_k = ?$

$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5\text{ kg} \times (2\text{ m/s})^2 = 1\text{ J}$

14. $m = 100\text{ kg}$ $h = 10\text{ m}$ $E_{pg} = ?$

$E_{pg} = mgh = 100\text{ kg} \times 9.80\text{ m/s}^2 \times 10\text{ m} = 9.80 \times 10^3\text{ J}$

Textbook, p. 18

15. Reference level chosen for the gravitational potential energy: the foot of the cliff. Therefore, at the start of the fall, $h = 50$ m; at the end of the fall, $h = 0$.

$E_k = \frac{1}{2}mv^2$ $E_{pg} = mgh$

a) At the start of the fall (at the top of the cliff), $v = 0 \Rightarrow E_k = 0$, therefore the kinetic energy is zero.

$h = 50\text{ m} \Rightarrow h$ is at a maximum, therefore the gravitational potential energy is at a maximum.

b) At a height of 25 m above the foot of the cliff, $h = 25\text{ m} \Rightarrow h$ corresponds to half of the initial height, therefore the gravitational potential energy is equal to half of the initial gravitational potential energy. The lost gravitational potential energy has been transformed into kinetic energy. The kinetic energy and the gravitational potential energy are therefore equal.

Textbook, p. 19

16. $E = 1.08\text{ MJ} = 1.08 \times 10^6\text{ J}$ $\Delta t = 5\text{ h} = 1.8 \times 10^4\text{ s}$
$P = ?$

$P = \frac{E}{\Delta t} = \frac{1.08 \times 10^6\text{ J}}{1.8 \times 10^4\text{ s}} = 60\text{ W}$

17. $P = 348\text{ W}$ $\Delta t = 2\text{ h} = 7.2 \times 10^3\text{ s}$ $E = ?$

$E = P\Delta t = 348\text{ W} \times 7.2 \times 10^3\text{ s} = 2.5 \times 10^6\text{ J}$
$= 2.5\text{ MJ}$

18. $P = 965\text{ W}$ $\Delta t = 4\text{ min} = 2.4 \times 10^2\text{ s}$ $E = ?$

$E = P\Delta t = 965\text{ W} \times 2.4 \times 10^2\text{ s} = 2.3 \times 10^5\text{ J}$
$= 0.23\text{ MJ}$

19. $P = 875\text{ W}$ $\Delta t = 10\text{ min} = 600\text{ s}$ $E = ?$

$E = P\Delta t = 875\text{ W} \times 600\text{ s} = 5.3 \times 10^5\text{ J} = 0.53\text{ MJ}$

UNIT 1

GEOMETRIC OPTICS

Chapter 1 Waves

Textbook, p. 23 to 40

PRACTICE MAKES PERFECT

Section 1.1 Characteristics of waves

Textbook, p. 28

1. $T = \dfrac{\text{Total time}}{\text{Number of cycles}} = \dfrac{3.00\ \text{s}}{375} = 0.008\,00\ \text{s}$

$f = \dfrac{\text{Number of cycles}}{\text{Total time}} = \dfrac{375}{3.00\ \text{s}} = 125\ \text{s}^{-1}$ or $125\ \text{Hz}$

2. $f = 2.00\ \text{Hz}$ $v = 5.40\ \text{m/s}$ $\lambda = ?$

$v = \lambda f \Rightarrow \lambda = \dfrac{v}{f} = \dfrac{5.40\ \text{m/s}}{2.00\ \text{s}^{-1}} = 2.70\ \text{m}$

3. $f = 440\ \text{Hz}$ $v = 350\ \text{m/s}$ $\lambda = ?$

$v = \lambda f \Rightarrow \lambda = \dfrac{v}{f} = \dfrac{350\ \text{m/s}}{440\ \text{s}^{-1}} = 0.795\ \text{m} = 79.5\ \text{cm}$

4. *a)* $f = \dfrac{\text{Number of cycles}}{\text{Total time}} = \dfrac{4800}{1\ \text{min}} = \dfrac{4800}{1\ \text{min}} \times \dfrac{1\ \text{min}}{60\ \text{s}}$

$f = 80\ \text{s}^{-1} = 80\ \text{Hz}$

b) $T = \dfrac{1}{f} = \dfrac{1}{80\ \text{s}^{-1}} = 0.013\ \text{s}$

5. *a)* According to the graph, the amplitude (A) is equal to 0.5 cm.

b) According to the graph, 3.5 cycles cover a length of 22 cm.

$\lambda = \dfrac{\text{Total length}}{\text{Number of cycles}} = \dfrac{22\ \text{cm}}{3.5} = 6.3\ \text{cm}$

c) $f = 20\ \text{Hz} = 20\ \text{s}^{-1}$ $\lambda = 6.3\ \text{cm} = 0.063\ \text{m}$ $v = ?$

$v = \lambda f = 0.063\ \text{m} \times 20\ \text{s}^{-1} = 1.3\ \text{m/s}$

6. *a)* All of the waves have a sinusoidal shape.

b) Wave E.

c) $T_E < T_D < T_C < T_B < T_A$

d) Since the frequency (f) is related to the period (T) according to the formula $f = 1/T$, the wave with the biggest period has the smallest frequency. As such, wave A has the smallest frequency.

e) $f_A < f_B < f_C < f_D < f_E$

Section 1.2 Light waves

Textbook, p. 35

1. For all questions, $T = \dfrac{\text{Total time}}{\text{Number of cycles}}$

a) $T = \dfrac{375\ \text{min}}{5} = 75\ \text{min} \times \dfrac{60\ \text{s}}{1\ \text{min}} = 4.5 \times 10^3\ \text{s}$

b) $T = \dfrac{6.7\ \text{s}}{10} = 0.67\ \text{s}$

c) $T = \dfrac{1\ \text{min}}{1000} = 0.001\ \text{min} \times \dfrac{60\ \text{s}}{1\ \text{min}} = 0.06\ \text{s}$

d) $T = \dfrac{57\ \text{s}}{68} = 0.84\ \text{s}$

2. For all questions, $f = \dfrac{\text{Number of cycles}}{\text{Total time}}$

a) $f = \dfrac{120}{2.0\ \text{s}} = 60\ \text{s}^{-1} = 60\ \text{Hz}$

b) $f = \dfrac{1200}{60\ \text{s}} = 20\ \text{s}^{-1} = 20\ \text{Hz}$

c) Total time $= 1.2\ \text{h} = 1.2\ \text{h} \times \dfrac{3600\ \text{s}}{1\ \text{h}} = 4.3 \times 10^3\ \text{s}$

$f = \dfrac{40}{1.2\ \text{h}} = \dfrac{40}{4.3 \times 10^3\ \text{s}} = 0.0093\ \text{s}^{-1} = 0.0093\ \text{Hz}$

d) $f = \dfrac{65}{48\ \text{s}} = 1.4\ \text{s}^{-1} = 1.4\ \text{Hz}$

3. *a)* $f = \dfrac{1}{T} = \dfrac{1}{4.5 \times 10^3\ \text{s}} = 2.2 \times 10^{-4}\ \text{s}^{-1} = 2.2 \times 10^{-4}\ \text{Hz}$

b) $f = \dfrac{1}{T} = \dfrac{1}{0.67\ \text{s}} = 1.5\ \text{s}^{-1} = 1.5\ \text{Hz}$

c) $f = \dfrac{1}{T} = \dfrac{1}{0.06\ \text{s}} = 17\ \text{s}^{-1} = 17\ \text{Hz}$

d) $f = \dfrac{1}{T} = \dfrac{1}{0.84\ \text{s}} = 1.2\ \text{s}^{-1} = 1.2\ \text{Hz}$

4. *a)* $T = \dfrac{1}{f} = \dfrac{1}{60\ \text{s}^{-1}} = 0.017\ \text{s}$

b) $T = \dfrac{1}{f} = \dfrac{1}{20\ \text{s}^{-1}} = 0.050\ \text{s}$

c) $T = \dfrac{1}{f} = \dfrac{1}{0.0093\ \text{s}^{-1}} = 108\ \text{s} = 1.8\ \text{min}$

d) $T = \dfrac{1}{f} = \dfrac{1}{1.4\ \text{s}^{-1}} = 0.71\ \text{s}$

5. *a)* Infrared radiation is located toward the lower frequencies.

   b) Ultraviolet radiation is located toward the higher frequencies.

6. d = circumference $= 2\pi R = 2\pi \times 6400$ km

   $d = 4.021 \times 10^4$ km

   $v = c = 3.00 \times 10^8$ m/s $= 3.00 \times 10^5$ km/s

   $\Delta t = ?$

   $$d = v \times \Delta t \Rightarrow \Delta t = \frac{d}{v} = \frac{4.021 \times 10^4 \text{ km}}{3.00 \times 10^5 \text{ km/s}} = 0.134 \text{ s}$$

7. $d = 150 \times 10^6$ km

   $v = c = 3.00 \times 10^8$ m/s $= 3.00 \times 10^5$ km/s

   $\Delta t = ?$

   $$d = v \times \Delta t \Rightarrow \Delta t = \frac{d}{v} = \frac{150 \times 10^6 \text{ km}}{3.00 \times 10^5 \text{ km/s}} = 500 \text{ s}$$

   $d = 8.33$ min

8. $f = 95.1$ MHz $= 95.1 \times 10^6$ Hz $= 9.51 \times 10^7$ s^{-1}

   $v = c = 3.00 \times 10^8$ m/s

   $\lambda = ?$

   $$v = \lambda \times f \Rightarrow \lambda = \frac{v}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{9.51 \times 10^7 \text{ s}^{-1}} = 3.15 \text{ m}$$

9. *a)* The region of umbra is in Area C (no light rays appear in this area).

   The regions of penumbra are in areas B and D (these regions receive light rays coming from part of the source but not from all its points).

   b) If the light source were a point source, there would not be any regions of penumbra, only a clearly delimited region of umbra (the region of umbra would be comprised of Area C and parts of areas B and D).

10. During hot weather, it is preferable to wear white clothing. White clothes appear to be white because they reflect all the colours in the visible light spectrum. They absorb very little light. Black clothes, on the other hand, strongly absorb all colours and reflect very little light. The absorbed light rays increase the temperature of black clothing.

Chapter 1
Waves

Textbook, p. 40

■ **1.** $f = 2$ MHz $= 2 \times 10^6$ Hz $= 2 \times 10^6$ s^{-1}

   $v = 1.5$ km/s $= 1500$ m/s $\lambda = ?$

   a) $v = \lambda \times f$

   $$\lambda = \frac{v}{f} = \frac{1500 \text{ m/s}}{2 \times 10^6 \text{ s}^{-1}} = 7.5 \times 10^{-4} \text{ m} = 0.75 \text{ mm}$$

   b) Since ultrasounds are mechanical waves, their velocity depends on the medium in which they travel. This means that the velocity of ultrasounds moving through air is different from their velocity when they move through heart tissue.

◆ **2.** $v_{sound} = 330$ m/s $\Delta t = 4.0$ s $d = ?$

   $d = v_{sound} \times \Delta t = 330 \text{ m/s} \times 4.0 \text{ s} = 1.3 \times 10^3$ m $= 1.3$ km

◆ **3.** *a)* According to Table 1 on page 31, this frequency corresponds to the colour yellow.

   b) $f = 5.2 \times 10^{14}$ Hz $= 5.2 \times 10^{14}$ s^{-1} $T = ?$

   $$T = \frac{1}{f} = \frac{1}{5.2 \times 10^{14} \text{ s}^{-1}} = 1.9 \times 10^{-15} \text{ s}$$

   c) It is possible to calculate its wavelength since we know that the speed of light in air is roughly equal
   $c = 3.00 \times 10^8$ m/s.

   $$v = \lambda \times f \Rightarrow \lambda = \frac{v}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.2 \times 10^{14} \text{ s}^{-1}}$$

   $$\lambda = 5.8 \times 10^{-7} \text{ m} \times \frac{1 \text{ nm}}{10^{-9} \text{ m}} = 580 \text{ nm}$$

   d) If this light wave reaches a shimmering surface, it will be reflected, meaning it will be sent back to the medium it came from.

■ **4.** The electromagnetic spectrum includes all the wave types comprised in an electric field and a magnetic field. Sound waves are mechanical waves that require a medium in which to travel. They are not comprised of a combination of two—magnetic and electric—fields. This is why sound waves do not figure in the electromagnetic spectrum.

★ **5.** *a)* A wave's period (T) corresponds to the time required for the wave to complete a cycle. According to the graph, $T = 4 \times 1.25\ \text{s} = 5.00\ \text{s}$.

b) $f = \frac{1}{T} = \frac{1}{5.00\ \text{s}} = 0.200\ \text{s}^{-1} = 0.200\ \text{Hz}$

c) It is not possible to determine the wavelength (λ). To do so would require the wave's velocity to be specified in the statement or the provision of a diagram of the wave.

d) According to the graph, $A = 2$ cm.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{5.00\ \text{s}} = 1.26\ \text{s}^{-1}$

Therefore: $y(t) = 2 \times \sin(1.26\ t)$

In this equation, the units y and t are centimetres (cm) and seconds (s), respectively.

Chapter 2 Reflection of Light

Textbook, p. 41 to 76

PRACTICE MAKES PERFECT

Section 2.1 Types of reflection

Textbook, p. 43

1. Specular reflection occurs on a smooth surface; parallel incident rays retain their parallel motion once reflected. Diffuse reflection occurs on rough surfaces; parallel incident rays are reflected in different directions.

2. Polishing a floor diminishes surface irregularities. The reflection, short of being completely specular, becomes less diffuse. Reflections on the unpolished sections remain diffuse and the floor stays matte.

3. The outer layer of glossy paper reflects a large part of incident light. The reflection is "fairly specular." If the light source, the paper and the eyes are positioned in such a way that the reflected beam is essentially directed toward the eyes, reading will be very difficult. In fact, the quantity of reflected light may dazzle the reading person or, at the very least, make it difficult to distinguish the light reflected by the printed pigments (characters, images) from the light reflected by the glossy paper.

Incident light that falls on matte paper is diffused in all directions, reducing the chances that the eyes receive excessive light.

4. A lake's surface is not smooth; reflections on its surface are not specular. Moreover, since the surface is constantly in motion, any images are rendered unstable.

5. *a)* Diffuse reflection.

b) (Almost) specular reflection.

c) Diffuse reflection.

d) (Almost) specular reflection.

e) Specular reflection.

6. *a)* The wavelength of green light (550 nm) is clearly greater than the size of surface irregularities (50 nm), making this a specular reflection.

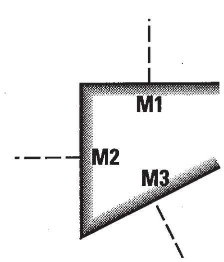
b) The wavelength of green light (550 nm) is smaller than the size of surface irregularities (2 µm or 2000 nm), making this a diffuse reflection.

Section 2.2 Geometry of reflection

Textbook, p. 45

1. B

2.



3. *a)* $\theta_i = 30°$

b) $\theta_{mirror} = 90° - 30° = 60°$

c) $\theta_i' = 30° + 20° = 50°$

d) $\theta'_{mirror} = 90° - 50° = 40°$

Section 2.3
Reflection on a plane mirror: laws of reflection

Textbook, p. 47

1. $\theta_i + \theta_r = 60° \quad \theta_i = ?$

According to the second law of reflection, $\theta_i = \theta_r$.
It follows that $2\theta_i = 60°$ and therefore that $\theta_i = 30°$.

2. $\theta_i = 20°$

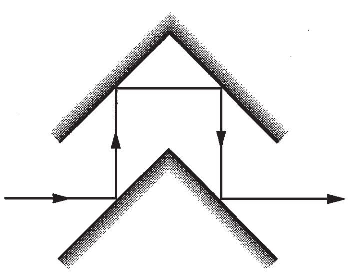
a) $\theta_r = ?$

According to the second law of reflection, $\theta_r = 20°$

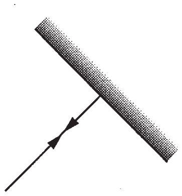
b) $\theta_{mirror} = 90° - 20° = 70°$

3. $\theta_i = 90° - 40° = 50°$ therefore $\theta_r = \theta_i = 50°$

4.



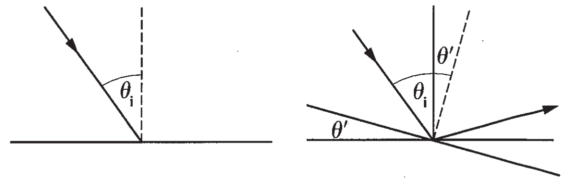
5.



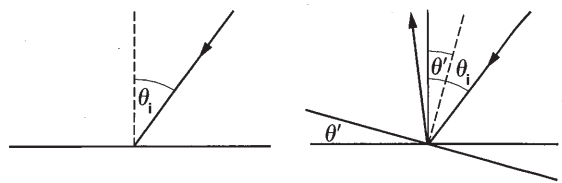
The ray reflects back on itself (the reflected ray merges with the incident ray) since the angle of incidence is zero and therefore the angle of reflection is zero as well.

6. The answer varies depending on how the normal and the incident ray are drawn.

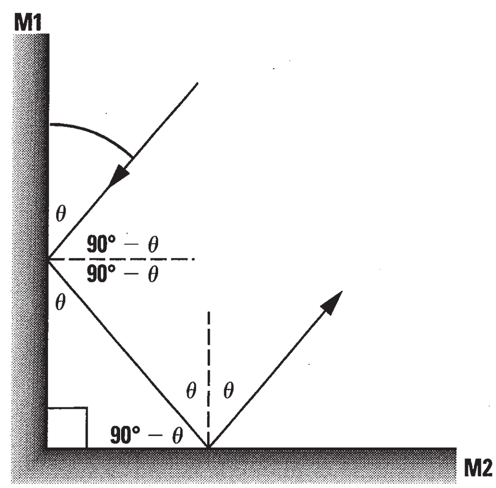
If the rotation of the mirror causes the normal to move away from the incident ray, the new angle of incidence becomes $\theta_{i2} = \theta_i + \theta'$ and therefore the angle of reflection will be $\theta_{r2} = \theta_i + \theta'$.



If the rotation of the mirror causes the normal to move closer to the incident ray, the new angle of incidence becomes $\theta_{i2} = \theta_i - \theta'$ (assuming that $\theta' < \theta_i$) and therefore the angle of reflection will be $\theta_{r2} = \theta_i - \theta'$.



7. Let us assume θ to be the angle between mirror M1 and the incident ray. The angle of incidence is equal to $90° - \theta$.

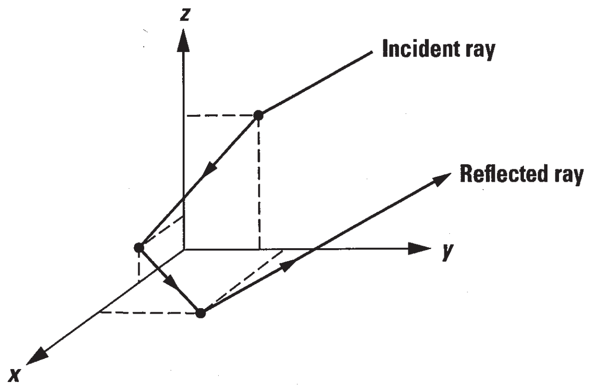


With each reflection, the reflected ray's direction obeys the second law of reflection. The ray reflected by M2 is at angle θ to a normal that is parallel to mirror M1. Since the incident ray is also at angle θ of mirror M1, the incident ray and the reflected ray are parallel.

8. With each reflection, the reflected ray's direction obeys the second law of reflection. This way, if the incident ray is perpendicular to one of the mirrors, it reflects back on itself and returns back to where it came from.

If the incident ray is parallel to one of the three mirrors, the ray can reflect in two mirrors in succession. As question 7 demonstrates, the ray returns toward its origin on a path parallel to the incident ray. In this case, the plane of incidence is perpendicular to the two mirrors.

Generally speaking, which is to say in any plane of incidence, a light ray undergoes three reflections, as indicated in the following figure.



Configuration of light rays in generalized three-dimensional cases

The reflected ray after the third reflection is parallel to the incident ray.

As such, in a retroreflector with cube corners, the reflected ray is always parallel to the incident ray, regardless of the direction of incidence.

Section 2.4
Reflection on spherical mirrors

Textbook, p. 54

1. 1: Principal axis 2: Centre of curvature (C) 3: Focal point (F) 4: Vertex (V) 5: Concave mirror (or converging mirror)

2. A spoon's interior is a concave (however not truly spherical) mirror, and its exterior is a convex mirror.

3. Let us assume θ to be the angle between the incident ray and the principal axis. The angle of reflection is also equal to θ. Therefore, the angle between the incident ray and the reflected ray is equal to 2θ.

4. $f = 20$ cm $R = ?$

   $R = 2f = 2 \times 20 \text{ cm} = 40 \text{ cm}$

5. With a parabolic mirror, all incident rays parallel to the principal axis after reflection converge exactly on the same point: the focal point. With a spherical mirror, the reflected rays do not converge exactly on the same point, and the focal point is less well defined.

6. Since the incident ray is parallel to the principal axis, it passes through the focal point after reflection.

   Since $R = 30$ cm, $f = \frac{R}{2} = \frac{30 \text{ cm}}{2} = 15$ cm.

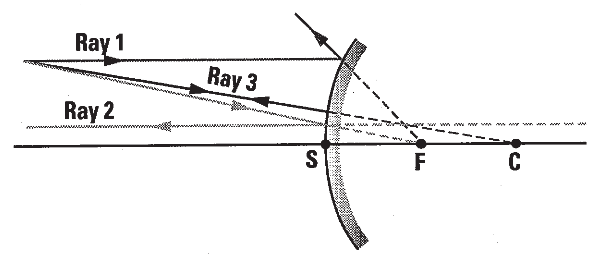
   The reflected ray will cross the principal axis 15 cm from the mirror's vertex.

7. A: The reflected ray passes through C (third principal ray).

   B: The reflected ray passes through F (first principal ray).

   C: The reflected ray is parallel to the principal axis (second principal ray).

   D: The reflected ray passes through F (first principal ray).

8. 

9. A: The reflected ray is parallel to the principal axis (second principal ray).

   B: The travelling reflected ray is directed at F (first principal ray).

   C: The travelling reflected ray is directed at C (third principal ray). As such, the ray is reflected back on itself.

   D: The travelling reflected ray is directed at F (first principal ray).

Section 2.5
Images

Textbook, p. 70

1. $d_o = 50$ m $h_o = ?$ $d_i = 20 \text{ cm} = 0.20$ m

   $h_i = 4.0 \text{ cm} = 0.040$ m

   $\frac{h_i}{h_o} = \frac{d_i}{d_o} \Rightarrow h_o d_i = h_i d_o \Rightarrow h_o = \frac{h_i d_o}{d_i}$

   $h_o = \frac{0.040 \text{ m} \times 50 \text{ m}}{0.20 \text{ m}} = 10 \text{ m}$

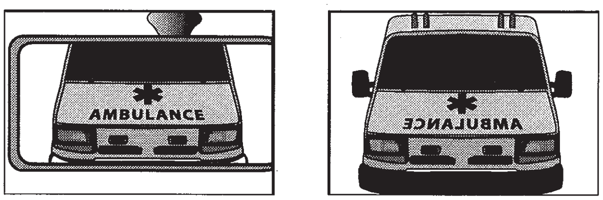
2. $h_o = 30 \text{ m} = 3000 \text{ cm}$ $\quad h_i = 1.5 \text{ cm}$ $\quad g = ?$

$$g = \frac{h_i}{h_o} = \frac{1.5 \text{ cm}}{3000 \text{ cm}} = 5.0 \times 10^{-4}$$

3. The image is virtual (it is necessary to look in the mirror to see it), upright and is the same height as the object.

4. 4 a.m.; 3:30 a.m.; 6 p.m. (or 6 a.m.).

5. The orientation of the image. The word "AMBULANCE" is written from right to left on the hood of the ambulance (frontal view) so that it can be read by someone looking at the image of the ambulance in the rear-view mirror of a car. Since the image provided in a rear-view mirror (plane mirror) is upright, the letter "A," located on the ambulance's left side, appears to be located on the left in the image so that the word can be read normally from left to right.



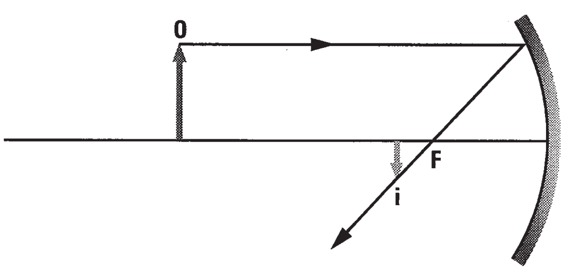
6. $\theta = 60°$ $\quad N = ?$

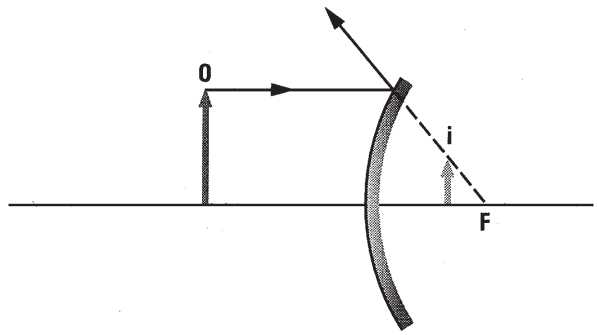
$$N = \left(\frac{360°}{\theta}\right) - 1 = \left(\frac{360°}{60°}\right) - 1 = 6 - 1 = 5$$

7. We know that:
   - the focal point is located on the principal axis;
   - an incident ray that is parallel to the principal axis (first principal ray) is reflected toward the focal point or in a direction originating from the focal point.

   Draw these rays or their extensions and determine the point where the reflected ray intersects the principal axis. This point of intersection is the focal point.

   Note that an incident ray originating from the tip of the object, after reflection, passes through the corresponding point at the tip of the image.





8. $f = \text{-}60.0 \text{ cm}$ $\quad d_o = 6 \text{ m} = 600 \text{ cm}$ $\quad h_o = 1.5 \text{ m}$

$d_i = ?$ $\quad h_i = ?$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{\text{-}60.0 \text{ cm}} - \frac{1}{600 \text{ cm}}$$

$$\frac{1}{d_i} = \frac{\text{-}10}{600 \text{ cm}} - \frac{1}{600 \text{ cm}} = \frac{\text{-}11}{600 \text{ cm}}$$

$$d_i = \frac{600 \text{ cm}}{\text{-}11} = \text{-}55 \text{ cm}$$

$$M = \frac{h_i}{h_o} = \text{-}\frac{d_i}{d_o} \Rightarrow h_i = \text{-}\frac{h_o d_i}{d_o}$$

$$h_i = -\frac{1.5 \text{ m} \times (\text{-}55 \text{ cm})}{6 \text{ m}} = 14 \text{ cm}$$

Since $d_i < 0$, the image is virtual, and since $h_i > 0$, the image is upright.

9. The rear-view mirror is convex therefore $f = \text{-}50 \text{ cm}$

$|M| = 0.10$ $\quad d_o = ?$ $\quad d_i = ?$

First the magnification sign needs to be determined. In the case of a convex mirror, the image is always upright (*see Table 4 on page 61*) and therefore $M = +0.10$.

$$M = \frac{h_i}{h_o} = \text{-}\frac{d_i}{d_o} = +0.10 \Rightarrow d_i = \text{-}0.10 \times d_o$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{\text{-}50 \text{ cm}} = \frac{1}{d_o} + \frac{1}{d_i}$$

The two equations above constitute a system of two equations with two unknowns, d_o and d_i, which need to be solved by one of the usual methods. For example, d_i in the term on the right in the second equation could be replaced with the expression of d_i provided in the first equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{\text{-}0.10 d_o} = \frac{1}{d_o} - \frac{1}{0.10 d_o} = \frac{0.10}{0.10 d_o} - \frac{1}{0.10 d_o}$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{-0.90}{0.10d_o}=\frac{-9}{d_o}$$

Therefore we obtain:

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\Rightarrow\frac{1}{-50\text{ cm}}=\frac{-9}{d_o}$$

$d_o = -9 \times (-50\text{ cm}) = 450\text{ cm} = 4.5\text{ m}$

and

$d_i = -0.10 \times d_o = -0.10 \times 4.5\text{ m} = -0.45\text{ m} = -45\text{ cm}$

Since $d_i < 0$, the image is virtual, which is consistent with the fact that the mirror is convex and the driver sees her image in the mirror.

10. Object: tooth $d_o = 15\text{ mm}$

$f = +20\text{ mm}$ $M = ?$

Since $M = -\frac{d_i}{d_o}$, d_i must be determined in order to deduce M.

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}$$

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{20\text{ mm}}-\frac{1}{15\text{ mm}}$$

$$=\frac{3}{60\text{ mm}}-\frac{4}{60\text{ mm}}=\frac{-1}{60\text{ mm}}$$

$d_i = -60\text{ mm}$

The image is virtual as the object (the tooth) is located between the focal point and the mirror. The dentist can see the tooth by looking into the mirror.

$$M=-\frac{d_i}{d_o}=-\frac{(-60\text{ cm})}{15\text{ mm}}=+4.0$$

Since $M > 0$, the image is upright.

11. Object: person $d_o = 2\text{ m}$ $R = ?$

If the image is three times larger than the object, the mirror must be concave, seeing how convex mirrors always provide a (virtual) image that is smaller than the object (*see Table 4 on page 61*).

Since M and d_o, are known, we can deduce d_i, and then f and R:

$$M=-\frac{d_i}{d_o}\Rightarrow d_i=-M\times d_o=-3\times 2\text{ m}=-6\text{ m}$$

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{2\text{ m}}+\frac{1}{-6\text{ m}}=\frac{3}{6\text{ m}}+\frac{-1}{6\text{ m}}=\frac{2}{6\text{ m}}$$

$$\frac{1}{f}=\frac{1}{3\text{ m}}\Rightarrow f=3\text{ m}$$

$R = 2f = 2 \times 3\text{ m} = 6\text{ m}$

Since the radius of curvature is positive, the mirror is in fact concave.

The person would see an image that is real and inverted. In this case, the solution is:

$M = -3$

$$M=\frac{-d_i}{d_o}\Rightarrow d_i=-M\times d_o=-(-3)\times 2\text{ m}=+6\text{ m}$$

$f = 1.5\text{ m}$

$R = 3\text{ m}$

12. The ornament constitutes a convex mirror.

Diameter $= 8\text{ cm} \Rightarrow |R| = 4\text{ cm} \Rightarrow |f| = 2\text{ cm}$

Since the mirror is convex, $R < 0$ and $f < 0$ according to sign convention, and therefore $f = -2\text{ cm}$.

The image is reduced by half. Since the mirror is convex, the image is upright (*see Table 4 on page 61*). Therefore, we write $M = +0.5$.

$d_o = ?$

$$M=\frac{h_i}{h_o}=-\frac{d_i}{d_o}=+0.5\Rightarrow d_i=-0.5\times d_o$$

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\Rightarrow\frac{1}{f}=\frac{1}{-2\text{ cm}}=\frac{1}{d_o}+\frac{1}{d_i}$$

The two equations above constitute a system of two equations with two unknowns, d_o and d_i. We can replace d_i in the term on the right in the second equation by the expression of d_i provided in the first equation:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{-0.5d_o}=\frac{1}{d_o}-\frac{1}{0.5d_o}=\frac{0.5}{0.5d_o}-\frac{1}{0.5d_o}$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{-0.5}{0.5d_o}=\frac{-1}{d_o}$$

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_1}\Rightarrow\frac{1}{-2\text{ cm}}=\frac{-1}{d_o}\Rightarrow d_o=-1\times(-2\text{ cm})=2\text{ cm}$$

Chapter 2 Reflection of Light

Textbook, p. 76

1. Let us assume β to be the angle between the reflected ray and the plane mirror.

$\beta = 90° - \theta_r$ and since $\theta_r = \theta_i$, therefore

$\beta = 90° - \theta_i$ and $\theta_i = 90° - \beta$.

2. a) If the centre of curvature is moved farther away, both R and f increase since $f = \frac{R}{2}$.

b) It becomes infinite.

c) A plane mirror.

3. a) M1: plane mirror; M2: convex (or diverging) mirror.

b) Since the angle of incidence is equal to the angle of reflection, the ray reflected by mirror M2 will pass through point B.

4. Mirror M1 shows an erroneous reflection: the angle of incidence (θ_i) is not equal to the angle of reflection (θ_r).

5. Using the second law of reflection, we discover that the ray reaches mirror M2 after it is reflected by mirrors M4 and M5.

6. Since virtual images form behind plane mirrors, mirrors give the impression that there is something behind them, thereby making the room appear larger.

7. First of all, when the person is beyond the centre of curvature, the image is real, inverted, smaller than the person, and between the centre and focal point.

When the person is at the centre of curvature, the image is real, inverted, the same size as the person, and at point C.

When the person is between the centre of curvature and the focal point, the image is real, inverted, larger than the person, and beyond the centre of curvature.

When the person is at the focal point, there is no image (or the image is infinite—which amounts to the same thing).

When the person is between the focal point and the mirror, the image is virtual, upright, larger than the person, behind the mirror, and farther away from the mirror than the person.

8. The person's face is at (or near) the spoon's focal point.

9. $f = -20$ cm $d_o = 50$ cm $h_o = 25$ cm $d_i = ?$ $h_i = ?$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-20\text{ cm}} - \frac{1}{50\text{ cm}}$$

$$= \frac{-5}{100\text{ cm}} - \frac{2}{100\text{ cm}} = \frac{-7}{100\text{ cm}}$$

$$d_i = \frac{-100\text{ cm}}{7} = -14\text{ cm}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$h_i = -\frac{d_i h_o}{d_o} = -\frac{(-14\text{ cm})(25\text{ cm})}{50\text{ cm}}$$

$$M = +7.0\text{ cm}$$

Since $d_i < 0$ and $h_i > 0$, the image is virtual and upright (and smaller than the object), as is usually the case with convex mirrors.

Chapter 3 Refraction of Light

Textbook, p. 77 to 94

PRACTICE MAKES PERFECT

Section 3.2 Index of refraction

Textbook, p. 81

1. $v = 2.50 \times 10^8$ m/s $c = 3.00 \times 10^8$ m/s $n = ?$

$$n = \frac{c}{v} = \frac{3.00 \times 10^8\text{ m/s}}{2.50 \times 10^8\text{ m/s}} = 1.20$$

2. $n = 1.92$ $c = 3.00 \times 10^8$ m/s $v = ?$

$$n = \frac{c}{v}$$

$$v = \frac{c}{n} = \frac{3.00 \times 10^8\text{ m/s}}{1.92} = 1.56 \times 10^8\text{ m/s}$$

3. $v_{air} = c = 3.00 \times 10^8$ m/s

$v_{zircon} = 1.56 \times 10^8$ m/s (*see question 2*)

$\Delta v = ?$

$\Delta v = v_{air} - v_{zircon} = 3.00 \times 10^8$ m/s $- 1.56 \times 10^8$ m/s

$\Delta v = 1.44 \times 10^8$ m/s

4. $c = 3.00 \times 10^8$ m/s $n = 1.50$ (glass) $v = ?$

$$n = \frac{c}{v}$$

$$v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50} = 2.00 \times 10^8 \text{ m/s}$$

5. Quartz: $d = 1.00$ m $n = 1.55$ $v = ?$ $\Delta t = ?$

$$n = \frac{c}{v}$$

$$v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.55} = 1.94 \times 10^8 \text{ m/s}$$

$$d = v \times \Delta t$$

$$\Delta t = \frac{d}{v} = \frac{1.00 \text{ m}}{1.94 \times 10^8 \text{ m/s}} = 5.15 \times 10^{-9} \text{ s}$$

6. The relative index of refraction is equal to the ratio between the indices of refraction of two transparent media:

$$n_{1\to2} = \frac{n_2}{n_1}$$

If the value of n_2 is lower than that of n_1, meaning if the ray passes from a more refractive to a less refractive medium, the ratio $\frac{n_2}{n_1}$ is less than 1.

7. The higher a medium's index of refraction (n), the greater its refractivity. As such, the least refractive medium is the one with the lowest index of refraction: medium B.

Section 3.3 Geometry of refraction

Textbook, p. 83

1. *a)* Given that the angle of refraction is smaller than the angle of incidence, the index of refraction of medium 2 is higher than that of medium 1 ($n_2 > n_1$).

b) The higher a medium's index of refraction (n) the greater its refractivity. Therefore, medium 2 is more refractive.

2. $n_1 = n_{glass} = 1.50$ et $n_2 = n_{water} = 1.33$

As such, $n_2 < n_1$: water is less refractive than glass. It follows that the refracted ray bends away from the normal and that the angle of refraction (θ_R) is greater than the angle of incidence (θ_i).

3. By definition, the relative index of refraction is written:

$$n_{1\to2} = \frac{n_2}{n_1}$$

If it is less than 1, we have $\frac{n_2}{n_1} < 1 \Rightarrow n_2 < n_1$

This means that medium 2 is less refractive than medium 1. It follows that the refracted ray bends away from the normal and therefore that the angle of refraction (θ_R) is greater than the angle of incidence (θ_i).

Section 3.4 Laws of refraction

Textbook, p. 86

1. *a)* 0.50 *b)* 0.87 *c)* 0.71 *d)* 0.218
e) 0.963 *f)* 0 *g)* 1.0 *h)* 0.34

2. *a)* $\theta = \sin^{-1}(0.342) = 20.0°$ *b)* 40.0°
c) 44.4° *d)* 19.5° *e)* 90.0°

3. $\theta_i = 60°$ $n_1 = 1.00$ (air) $\theta_R = ?$

$n_2 = 2.42$ (diamond)

$$n_1 \sin\theta_i = n_2 \sin\theta_R$$

$$\sin\theta_R = \frac{n_1 \sin\theta_i}{n_2} = \frac{1.00 \times \sin 60°}{2.42} = 0.36$$

$$\theta_R = \sin^{-1}(0.36) = 21°$$

4. $\theta_i = ?$ $n_1 = 1.00$ (air) $\theta_R = 45°$ $n_2 = 1.30$

$$n_1 \sin\theta_i = n_2 \sin\theta_R$$

$$\sin\theta_i = \frac{n_2 \sin\theta_R}{n_1} = \frac{1.30 \times \sin 45°}{1.00} = 0.92$$

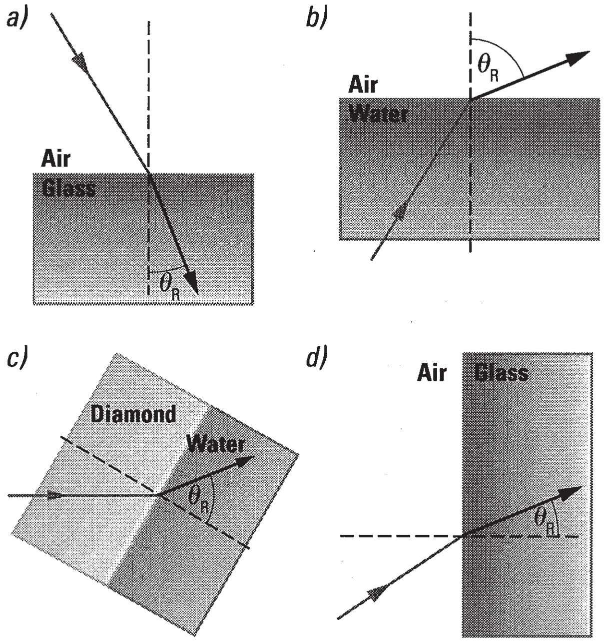
$$\theta_i = \sin^{-1}(0.92) = 67°$$

5. $\theta_i = 50°$ $n_1 = 1.00$ (air) $\theta_R = 40°$ $n_2 = ?$

$$n_1 \sin\theta_i = n_2 \sin\theta_R$$

$$n_2 = \frac{n_1 \sin\theta_i}{\sin\theta_R} = \frac{1.00 \times \sin 50°}{\sin 40°} = 1.2$$

6. The objective of this exercise is to determine whether the refracted ray bends toward or away from the normal.



7. $\theta_i = 30°$ $n_1 = 1.00$ (air) $\theta_R = ?$

For each of the media, n_2 is provided. We can determine θ_R using the second law of refraction:

$$n_1 \sin \theta_i = n_2 \sin \theta_R \Rightarrow \sin \theta_R = \frac{n_1 \sin \theta_i}{n_2}$$

a) $\sin \theta_R = 0.38 \Rightarrow \theta_R = \sin^{-1}(0.38) = 22°$

b) $\sin \theta_R = 0.21 \Rightarrow \theta_R = \sin^{-1}(0.21) = 12°$

c) $\sin \theta_R = 0.37 \Rightarrow \theta_R = \sin^{-1}(0.37) = 22°$

d) $\sin \theta_R = 0.26 \Rightarrow \theta_R = \sin^{-1}(0.26) = 15°$

8. $\theta_i = ?$ $\theta_R = 10°$

In each case, n_1 and n_2 are provided. We can determine θ_i using the second law of refraction:

$$n_1 \sin \theta_i = n_2 \sin \theta_R \Rightarrow \sin \theta_i = \frac{n_2 \sin \theta_R}{n_1}$$

a) $n_1 = 2.42$ $n_2 = 1.00 \Rightarrow \sin\theta_i = 0.072$

$\Rightarrow \theta_i = \sin^{-1}(0.072) = 4.1°$

b) $n_1 = 1.00$ $n_2 = 2.42 \Rightarrow \sin\theta_i = 0.420$

$\Rightarrow \theta_i = \sin^{-1}(0.420) = 25°$

c) $n_1 = 1.00$ $n_2 = 1.33 \Rightarrow \sin\theta_i = 0.231$

$\Rightarrow \theta_i = \sin^{-1}(0.231) = 13°$

d) $n_1 = 1.33$ $n_2 = 2.42 \Rightarrow \sin\theta_i = 0.316$

$\Rightarrow \theta_i = \sin^{-1}(0.316) = 18°$

9. In each case, θ_i, n_1 ($n_1 = 1.00$) and θ_R are provided. We can determine n_2 using the second law of refraction:

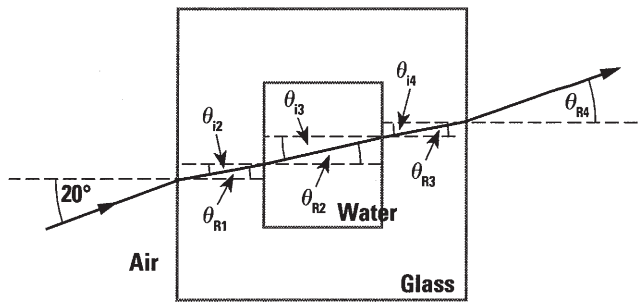
$$n_1 \sin \theta_i = n_2 \sin \theta_R \Rightarrow n_2 = \frac{n_1 \sin \theta_i}{\sin \theta_R}$$

a) $\theta_i = 40°$ $n_1 = 1.00$ $\theta_R = 30° \Rightarrow n_2 = 1.3$

b) $\theta_i = 30°$ $n_1 = 1.00$ $\theta_R = 12° \Rightarrow n_2 = 2.4$

c) $\theta_i = 77°$ $n_1 = 1.00$ $\theta_R = 50° \Rightarrow n_2 = 1.3$

10. Viewed from above, the aquarium (assumed to have parallel sides) appears as in the following diagram.



Air → glass interface: $\theta_{i1} = 20°$ $n_1 = 1.00$

$\theta_{R1} = ?$ $n_2 = 1.50$

$n_1 \sin \theta_{i1} = n_2 \sin \theta_{R1}$

$$\sin \theta_{R1} = \frac{n_1 \sin \theta_{i1}}{n_2} = \frac{1.00 \sin 20°}{1.50} = 0.228$$

$\theta_{R1} = 13.2°$

Glass → water interface: $\theta_{i2} = ?$ $n_1 = 1.50$

$\theta_{R2} = ?$ $n_2 = 1.33$

Since the air-glass and glass-water interfaces are parallel, angles θ_{R1} and θ_{i2} are alternate interior angles. As such, $\theta_{i2} = \theta_{R1} = 13.2°$.

$n_1 \sin \theta_{i2} = n_2 \sin \theta_{R2}$

$$\sin \theta_{R2} = \frac{n_1 \sin \theta_{i2}}{n_2} = \frac{1.50 \sin 13.2°}{1.33} = 0.258$$

$\theta_{R2} = 14.9°$

When the ray passes from glass to water (a less refractive medium than glass), it bends away from the normal.

Water → glass interface: $\theta_{i3} = 14.9°$ $n_1 = 1.33$

$\theta_{R3} = ?$ $n_2 = 1.50$

$n_1 \sin \theta_{i3} = n_2 \sin \theta_{R3}$

$$\sin \theta_{R3} = \frac{n_1 \sin \theta_{i3}}{n_2} = \frac{1.33 \sin 14.9°}{1.50} = 0.228$$

$\theta_{R3} = 13.2°$

Glass → air interface: $\theta_{i4} = 13.2°$ $n_1 = 1.50$
$\theta_{R4} = ?$ $n_2 = 1.00$

$n_1 \sin\theta_{i4} = n_2 \sin\theta_{R4}$

$$\sin\theta_{R4} = \frac{n_1 \sin\theta_{i4}}{n_2} = \frac{1.50 \sin 13.2°}{1.00} = 0.343$$

$\theta_{R4} = 20°$

The emerging ray travels in a direction parallel to that of the incident ray (however with a slight refraction).

11. $v = 2.67 \times 10^8$ m/s $c = 3.00 \times 10^8$ m/s $n = ?$

$$n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{2.67 \times 10^8 \text{ m/s}} = 1.12$$

This index of refraction is lower than that of water ($n = 1.33$).

12. A ray of light passing from air is refracted at the water's surface and reaches the diver's eyes.

$\theta_i = ?$ $n_1 = 1.00$ $\theta_R = 25°$ $n_2 = 1.33$

$n_1 \sin\theta_i = n_2 \sin\theta_R$

$$\sin\theta_i = \frac{n_2 \sin\theta_R}{n_1} = \frac{1.33 \times \sin 25°}{1.00} = 0.56$$

$\theta_i = 34°$

13. $\theta_i = 30°$ $n_1 = 1.33$ $\theta_R = ?$ $n_2 = 1.00$

$n_1 \sin\theta_i = n_2 \sin\theta_R$

$$\sin\theta_R = \frac{n_1 \sin\theta_i}{n_2} = \frac{1.33 \sin 30°}{1.00} = 0.67$$

$\theta_R = 42°$

Section 3.5
Total internal reflection

Textbook, p. 88

1. Medium 1: glass $n_1 = 1.50$ Medium 2: air $n_2 = 1.00$

The critical angle corresponds to the angle of incidence when the angle of refraction is 90°:

$n_1 \sin\theta_c = n_2 \sin\theta_R = n_2 \sin 90° = n_2$

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.00}{1.50} = 0.667$$

$\theta_c = 41.8°$

2. Medium 1: $\theta_c = 40.5°$ $n_1 = ?$
Medium 2: air $\theta_R = 90°$ $n_2 = 1.00$

$n_1 \sin\theta_c = n_2 \sin 90° = n_2$

$$n_1 = \frac{n_2}{\sin\theta_c} = \frac{1.00}{\sin 40.5°} = 1.54$$

3. Total internal reflection can only occur if light passes from one medium to another medium with a lower index of refraction. In an aquarium, total internal reflection can occur when light passes from glass to water (but not from water to glass).

4. This phenomenon occurs as a result of the refraction and total internal reflection of the light at the surface. Consider a ray that passes from air to water: its maximum angle of refraction for a maximum angle of incidence of 90° is obtained by:

$n_1 \sin\theta_i = n_2 \sin\theta_R$

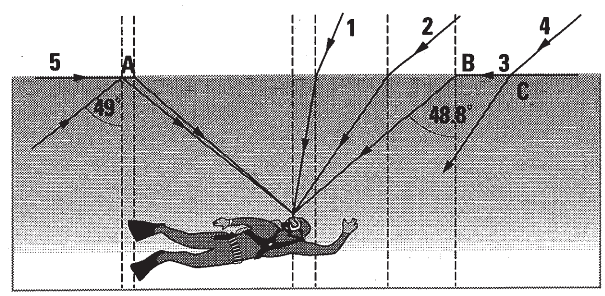
$$\sin\theta_R = \frac{n_1 \sin\theta_i}{n_2} = \frac{1.00 \times \sin 90°}{1.33} = 0.752$$

$\theta_R = 48.8°$

This angle is also the critical angle for a ray that passes from water to air:

$n_1 \sin\theta_c = n_2 \sin 90°$

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.00}{1.33} = 0.752 \Rightarrow \theta_c = 48.8°$$



Let us analyze the pathways of the different rays illustrated above. All incident rays penetrate the water. Among those illustrated, rays 1, 2 and 3 reach the diver, but not ray 4. Rays that penetrate the water beyond point B cannot be perceived by the diver; this peripheral area appears markedly darker than the area bounded by points A and B, where the rays coming from the air reach the diver. The area between A and B, which forms a (two-dimensional) circle at the surface, appears brighter to the diver's eye than the rest of the water's surface.

The rays reaching the diver's eyes that come from the peripheral area, beyond points A and B, are rays travelling through water (arriving from below, for example) after being reflected on the surface by total internal reflection. As such, the peripheral region appears like a mirror. This region appears dark because the light rays reflected on the surface are of low intensity (less bright in water). The circular surface between A and B above the diver appears to be a "hole."

5. *a)* $n_1 = 1.68 \quad \theta_c = ? \quad n_2 = 1.00$

$n_1 \sin \theta_c = n_2 \sin 90°$

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1.00}{1.68} = 0.595 \Rightarrow \theta_c = 36.5°$$

b) $n_1 = ? \quad \theta_c = 40° \quad n_2 = 1.00$

$n_1 \sin \theta_c = n_2 \sin 90°$

$$n_1 = \frac{n_2}{\sin \theta_c} = \frac{1.00}{\sin 40°} = 1.56$$

6. *a)* Refraction at the water-glass interface:

$n_1 = 1.33 \quad \theta_i = 30° \quad n_2 = 1.50 \quad \theta_R = ?$

$n_1 \sin \theta_i = n_2 \sin \theta_R$

$$\sin \theta_R = \frac{n_1 \sin \theta_i}{n_2} = \frac{1.33 \times \sin 30°}{1.50} = 0.443$$

$\theta_R = 26.3°$

Refraction at the glass-air interface:

$n_1 = 1.50 \quad \theta_{i2} = 26.3° \quad n_2 = 1.00 \quad \theta_{R2} = ?$

$n_1 \sin \theta_{i2} = n_2 \sin \theta_{R2} \qquad \sin \theta_{R2} = \dfrac{n_1 \sin \theta_{i2}}{n_2}$

$$\sin \theta_{R2} = \frac{1.50 \times \sin 26.3°}{1.00} = 0.665$$

$\theta_{R2} = 41.7°$

b) Refraction at the water-glass interface:

$n_1 = 1.33 \quad \theta_i = 52° \quad n_2 = 1.50 \quad \theta_R = ?$

$n_1 \sin \theta_i = n_2 \sin \theta_R$

$$\sin\theta_R = \frac{n_1 \sin \theta_i}{n_2} = \frac{1.33 \times \sin 52°}{1.50} = 0.699$$

$\theta_R = 44.3°$

Refraction at the glass-air interface:

$n_1 = 1.50 \quad \theta_{i2} = 44.3° \quad n_2 = 1.00 \quad \theta_{R2} = ?$

$n_1 \sin \theta_{i2} = n_2 \sin \theta_{R2}$

$$\sin \theta_{R2} = \frac{n_1 \sin \theta_{i2}}{n_2} = \frac{1.50 \times \sin 44.3°}{1.00} = 1.05$$

Since the value of the sine function is greater than 1, the situation is impossible and there is total internal reflection; the light ray does not emerge from the aquarium. In actual fact, when light passes from glass to air, the critical angle is 41.8° (*see question 1*). In this case, $\theta_{i2} = 44.3°$ and exceeds the critical angle: there is total internal reflection.

Chapter 3
Refraction of Light

Textbook, p. 93 and 94

1. Angle of incidence (θ_i): 7
Normal: 2
Refracted ray: 4
Angle of reflection (θ_r): 6
Incident ray: 3
Angle of refraction (θ_R): 10
Reflected ray: 1
Interface: 12

2. It is understood that the incident light ray is travelling through air therefore $n_1 = 1.00$ and $n_2 = 1.50$. In all three cases, the angle of refraction is calculated with

$$n_1 \sin \theta_i = n_2 \sin \theta_R \Rightarrow \sin\theta_R = \frac{n_1 \sin \theta_i}{n_2}.$$

And we obtain:

a) 0° *b)* 19° *c)* 35°

3. *a)* $\theta_i = 30°$

It is understood that the incident light ray travels through air; therefore, $n_1 = 1.00$.

The index of refraction of glass depends on the wavelength:

$n_{2red} = 1.52 \qquad n_{2violet} = 1.54$

For the red ray:

$n_1 \sin \theta_i = n_2 \sin \theta_{Rr}$

$$\sin\theta_{Rr} = \frac{n_1 \sin\theta_i}{n_{2r}} = \frac{1.00 \times \sin 30°}{1.52} = 0.329$$

$\theta_{Rr} = 19.2°$

For the violet ray:

$n_1 \sin\theta_i = n_2 \sin\theta_{Rv}$

$$\sin\theta_{Rv} = \frac{n_1 \sin\theta_i}{n_{2v}} = \frac{1.00 \times \sin 30°}{1.54} = 0.325$$

$\theta_{Rv} = 18.9°$

b) $\theta_{ir} = 19.2°$ $n_{1r} = 1.52$ $\theta_{iv} = 18.9°$ $n_{1v} = 1.54$

$\theta_{Rr} = ?$ $\theta_{Rv} = ?$ $n_2 = 1.00$

For the red ray:

$n_{1r} \sin\theta_{ir} = n_2 \sin\theta_{Rr}$

$$\sin\theta_{Rr} = \frac{n_{1r} \sin\theta_{ir}}{n_2} = \frac{1.52 \times \sin 19.2°}{1.00} = 0.50$$

$\theta_{Rr} = 30°$

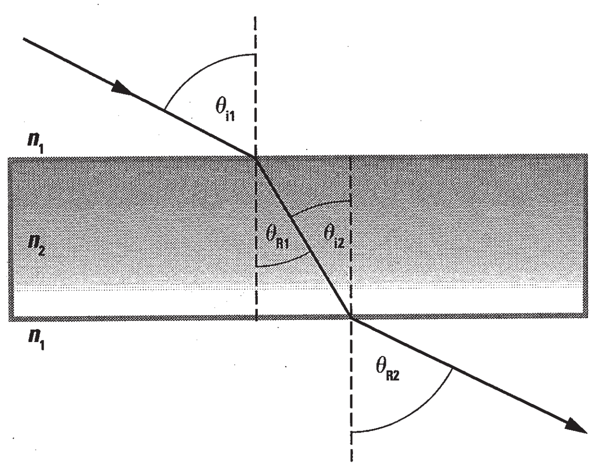
For the violet ray:

$n_{1v} \sin\theta_{iv} = n_2 \sin\theta_{Rv}$

$$\sin\theta_{Rv} = \frac{n_{1v} \sin\theta_{iv}}{n_2} = \frac{1.54 \times \sin 18.9°}{1.00} = 0.50$$

$\theta_{Rv} = 30°$

4. Let us assume an incident ray is travelling through air (n_1) and reaches, with an angle of incidence θ_{i1}, a sheet of glass with an index n_2. The angle of refraction at the first interface is θ_{R1}.



At the first interface: $n_1 \sin\theta_{i1} = n_2 \sin\theta_{R1}$ (1)

At the second interface: $n_2 \sin\theta_{i2} = n_1 \sin\theta_{R2}$ (2)

Since the two normals are parallel (because they are perpendicular to two parallel sides), θ_{R1} and θ_{i2} are alternate interior angles and therefore, $\theta_{i2} = \theta_{R1}$. The second equation therefore becomes: $n_2 \sin\theta_{R1} = n_1 \sin\theta_{R2}$ (3).

If we compare equations (1) and (3), we can write: $n_1 \sin\theta_{i1} = n_1 \sin\theta_{R2}$, and it follows that $\theta_{i1} = \theta_{R2}$. Since these angles are defined in relation to normals that are parallel to each other, the incident and emerging rays are parallel as well.

5. For the violet light: $n_v = 1.53$

For the red light: $n_r = 1.51$

$$n_v = \frac{c}{v_v} \Rightarrow v_v = \frac{c}{n_v} = \frac{3.00 \times 10^8 \text{ m/s}}{1.53} = 1.96 \times 10^8 \text{ m/s}$$

$$n_r = \frac{c}{v_r} \Rightarrow v_r = \frac{c}{n_r} = \frac{3.00 \times 10^8 \text{ m/s}}{1.51} = 1.99 \times 10^8 \text{ m/s}$$

6. First the index of refraction of water has to be determined.

$$v = \frac{3}{4}c \Rightarrow n = \frac{c}{v} = \frac{c}{\frac{3}{4}c} = \frac{4}{3} = 1.33$$

$\theta_i = 10°$ $n_1 = 1.00$ $\theta_R = ?$ $n_2 = 1.33$

$n_1 \sin\theta_i = n_2 \sin\theta_R$

$$\sin\theta_R = \frac{n_1 \sin\theta_i}{n_2} = \frac{1.00 \times \sin 10°}{1.33} = 0.13$$

$\theta_R = 7.5°$

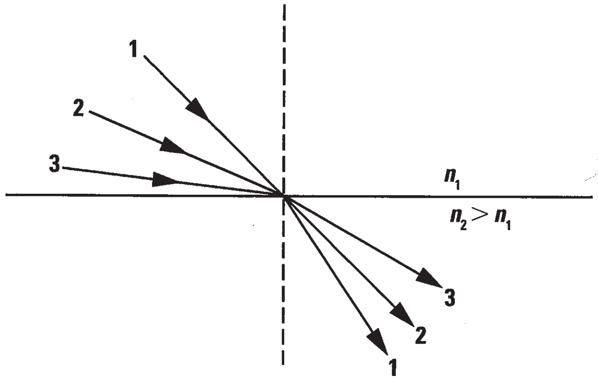
7. It is understood that a "less dense medium," is a medium with a lower index of refraction. When a ray passes from a less dense medium to a more dense medium, we need to consider that $n_1 < n_2$. In this case, the angle of incidence is always greater than the angle of refraction. Therefore, θ_R cannot possibly reach 90°.

Otherwise, more formally speaking, considering the algebraic expression of the law of refraction:

$$n_1 \sin\theta_i = n_2 \sin\theta_R$$

we note that if $n_1 < n_2$, in order to respect the equation, it is necessary that $\sin\theta_i > \sin\theta_R$, and that $\theta_i > \theta_R$. It is therefore impossible for θ_R to reach 90°, and so there is no critical situation or total internal reflection.

The following diagram illustrates the situation. We observe that regardless of the angle of incidence the refracted ray always exists.



◆ **8.** Light rays are refracted when they pass through the cornea (surface of the eye). Human eyes work in such a way that the cornea-crystalline lens optical system focuses the rays in order to produce a clear image on the retina when the eye is in air (*see Section 5.2 on page 134 in the textbook*).

If the eye is in water, the refraction of the cornea is much less pronounced, since we have $n_1 =$ 1.33 instead of $n_1 = 1.00$. The eye can no longer deviate enough rays to focus them on the retina, and images become blurred. The diving mask or swimming goggles make it possible to insert a layer of air in front of the eye and thereby restore the eye's normal optical conditions (the refraction occurs from air to eye, just like when we are out of the water).

◆ **9.** $\theta_i = ?$ $n_1 = 1.00$ $\theta_R = ?$ $n_2 = 1.33$

First θ_R needs to be determined, based on geometric information:

$$\tan \theta_R = \frac{\text{opposite}}{\text{adjacent}} = \frac{1.0\text{ m}}{1.2\text{ m}} = 0.83 \Rightarrow \theta_R = 40°$$

and then the law of refraction needs to be applied:

$$n_1 \sin \theta_i = n_2 \sin \theta_R \qquad \sin \theta_i = \frac{n_2 \sin \theta_R}{n_1}$$

$$\sin \theta_i = \frac{1.33 \times \sin 40°}{1.00} = 0.85 \qquad \theta_i = 59°$$

The incident ray must form an angle of 59° with the pool wall.

◆ **10.** The law of refraction needs to be applied at each interface while considering that the angle of incidence at one interface is equal to the angle of refraction at the preceding interface, as the angles are alternate interior angles.

Glass-carbon disulfide interface:

$\theta_i = 10°$ $n_1 = 1.50$ $\theta_R = ?$ $n_2 = 1.63$

$$n_1 \sin \theta_i = n_2 \sin \theta_R \qquad \sin \theta_R = \frac{n_1 \sin \theta_i}{n_2}$$

$$\sin \theta_R = \frac{1.50 \times \sin 10°}{1.63} = 0.160 \qquad \theta_R = 9.21°$$

Carbon disulfide-water interface:

$\theta_i = 9.21°$ $n_1 = 1.63$ $\theta_R = ?$ $n_2 = 1.33$

$$n_1 \sin \theta_i = n_2 \sin \theta_R \Rightarrow \sin \theta_R = \frac{n_1 \sin \theta_i}{n_2}$$

$$\sin \theta_R = \frac{1.63 \times \sin 9.21°}{1.33} = 0.196 \Rightarrow \theta_R = 11.3°$$

Water-oleic acid interface:

$\theta_i = 11.3°$ $n_1 = 1.33$ $\theta_R = ?$ $n_2 = 1.43$

$$n_1 \sin \theta_i = n_2 \sin \theta_R \qquad \sin \theta_R = \frac{n_1 \sin \theta_i}{n_2}$$

$$\sin \theta_R = \frac{1.33 \times \sin 11.3°}{1.43} = 0.182 \qquad \theta_R = 10.5°$$

Oleic acid-air interface:

$\theta_i = 10.5°$ $n_1 = 1.43$ $\theta_R = ?$ $n_2 = 1.00$

$$n_1 \sin \theta_i = n_2 \sin \theta_R \qquad \sin \theta_R = \frac{n_1 \sin \theta_i}{n_2}$$

$$\sin \theta_R = \frac{1.43 \times \sin 10.5°}{1.00} = 0.261 \qquad \theta_R = 15.1°$$

◆ **11.** The critical angle can serve to deduce the index of refraction. In both cases, $n_2 = 1.00$.

Medium A: $\theta_{cA} = 27°$

$$n_{1A} \sin \theta_{cA} = n_2 \sin 90°$$

$$n_{1A} = \frac{n_2}{\sin \theta_{cA}} = \frac{1.00}{\sin 27°} = 2.20$$

$$n_A = \frac{c}{v_A}$$

$$v_A = \frac{c}{n_A} = \frac{3.00 \times 10^8\text{ m/s}}{2.20} = 1.36 \times 10^8\text{ m/s}$$

Medium B: $\theta_{cB} = 32°$

$$n_{1B} \sin \theta_{cB} = n_2 \sin 90° \Rightarrow n_{1B} = \frac{n_2}{\sin \theta_{cB}} = \frac{1.00}{\sin 32°} = 1.89$$

$$n_B = \frac{c}{v_B} \Rightarrow v_B = \frac{c}{n_B} = \frac{3.00 \times 10^8 \text{ m/s}}{1.89} = 1.59 \times 10^8 \text{ m/s}$$

Light travels faster in medium B.

◆ **12.** *a)* Given that the incident ray arrives at a perpendicular angle to side AC, $\theta_i = 0$, therefore $\theta_R = 0$. The ray continues in a straight line.

b) The ray is completely reflected on side AB of prism P1, since the angle of incidence (θ_i) at this side is such that total internal reflection occurs ($\theta_i > \theta_c$).

c) The angle of reflection (θ_r) is equal to the angle of incidence (θ_i). The prism is an isosceles prism, meaning that angle $\angle CAB = 45°$. It can be deduced that for side AB, $\theta_i = 45°$.

As such, θ_r is equal to 45°.

d) For the ray to undergo total internal reflection, it is necessary that $\theta_i \geq \theta_c$. In this figure $\theta_i = 45°$. It is therefore necessary that $\theta_c \leq 45°$.

Since $n_1 \sin \theta_c = n_2 \sin 90°$, it follows

that $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$.

Therefore, $\sin^{-1}\left(\frac{n_2}{n_1}\right) \leq 45°$ and, since the sine function increases (when the angle increases, its sine value increases) when applying the sine function on each side of the equation, we obtain $\frac{1.00}{n_1} \leq \sin 45°$.

By isolating n_1, we obtain $n_1 \geq \frac{1.00}{\sin 45°}$. As such, it is necessary that $n_1 \geq 1.41$.

For there to be total internal reflection with an angle of incidence (θ_i) equal to 45°, the prisms' index of refraction (n) must be greater than 1.41.

Chapter 4 Lenses

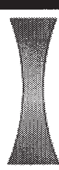
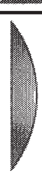
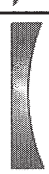
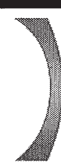
Textbook, p. 95 to 130

PRACTICE MAKES PERFECT

Section 4.1 Different types of lenses

Textbook, p. 98

1.

Lens	Type	Converging/ diverging lens	Symbol
	Biconcave lens	Diverging lens	
	Planoconvex lens	Converging lens	
	Planoconcave lens	Diverging lens	
	Negative meniscus lens	Diverging lens	
	Biconvex lens	Converging lens	
	Positive meniscus lens	Converging lens	

2. *a)* Converging lenses are thicker at the centre than at the edges while diverging lenses are thinner at the centre than at the edges.

b) A converging lens deviates the incident rays that are parallel to the principal axis (P) so that after having passed through the lens, the rays converge toward a single point on the other side of the lens.

A diverging lens deviates the incident rays that are parallel to the principal axis so that after having passed through the lens, the rays travel away from one another.

3. Characteristics of converging spherical lenses

Converging lens	First surface	Second surface
Biconvex	Convex spherical	Convex spherical
Positive meniscus	Concave spherical	Convex spherical
Planoconvex	Flat	Convex spherical

Obviously, because a lens can be inverted, a positive meniscus lens can have a convex spherical form for a first surface and a concave spherical form for a second surface. The same observation applies to the planoconvex lens.

4. Characteristics of diverging spherical lenses

Diverging lens	First surface	Second surface
Planoconcave	Flat	Concave spherical
Biconcave	Concave spherical	Concave spherical
Negative meniscus	Convex spherical	Concave spherical

5. They are diverging.

6. The convex surface has a more pronounced curvature than the concave surface (its radius of curvature is therefore smaller).

7. *a)* *Converging* lenses are thicker at the centre than at the edges.

   b) Diverging lenses are *thinner* at the centre than at the edges.

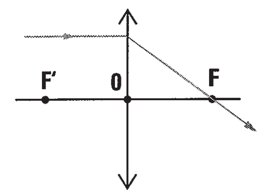
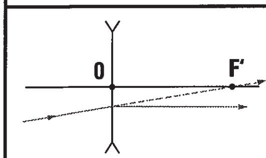
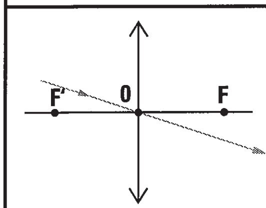
8. Individual answer. At minimum, the student must mention the fact that in a converging lens, rays that are parallel to the principal axis (P) strike one of the surfaces of the lens and converge to a single point on the opposite side of the lens. The light will therefore be more concentrated, and more intense, in the area near this point.

9. The student must draw a planoconvex lens.

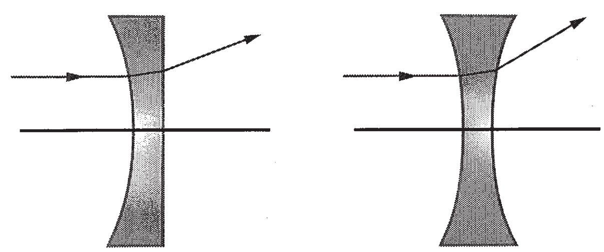
Section 4.2 Refraction in lenses

Textbook, p. 105

1.

Ray diagram	Type of ray	Converging/ diverging lens
	First principal ray	Converging lens
	Second principal ray	Diverging lens
	Third principal ray	Converging lens

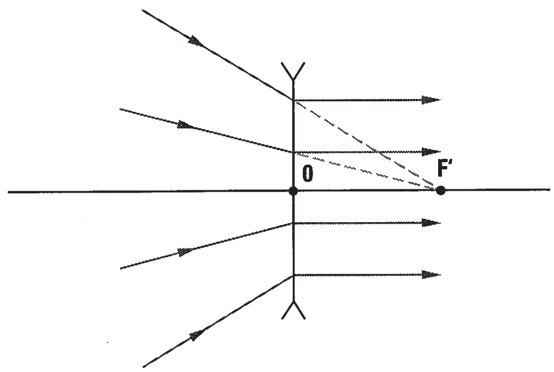
2. Biconcave lenses deviate the light more.



For the incident ray on the two lenses illustrated above, the angle of incidence is the same for the first surface and, therefore, so is the angle of refraction. However, on the second surface, the angle of incidence is greater for the biconcave lens and, therefore, so is the angle of refraction. Therefore, the deviation of the ray is greater for the biconcave lens than for the planoconcave lens.

3. 1: Principal axis (P)
   2: Secondary focal point (F')
   3: Converging lens
   4: Optical centre (O)
   5: Principal focal point (F)

4. *a)* The lens is diverging.
   b) Point F' is called the secondary focal point.
   c) Any incident ray that points in the direction of the secondary focal point (F') of a diverging lens is refracted parallel to the principal axis (P) of the lens (second principal ray).

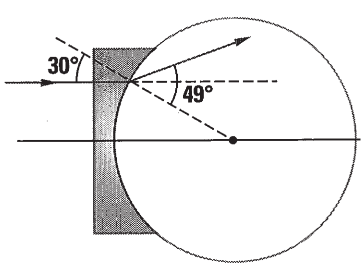


5. *a)* It is a planoconcave lens.
   b) This line is the principal axis (P) of the lens.
   c) At the first interface, $\theta_i = 0$ and, therefore, $\theta_R = 0$: the ray continues in a straight line.

   At the second interface: $\theta_i = 30°$ $n_1 = 1.50$
   $\theta_R = ?$ $n_2 = 1.00$

$$n_1 \sin \theta_i = n_2 \sin \theta_R \Rightarrow \sin \theta_R = \frac{n_1 \sin \theta_i}{n_2}$$

$$\sin \theta_R = \frac{1.50 \times \sin 30°}{1.00} = 0.75 \Rightarrow \theta_R = 49°$$



   d) Because the emerging ray bends away from the principal axis (with an angle equal to $49 - 30 = 19°$), the lens is diverging.

Section 4.3 Optical power of lenses

Textbook, p. 112

1. $n_{glass} = 1.50$ $n_{diamond} = 2.42$

   Since $P = \frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, a greater index of refraction implies that optical power P is higher (considering identical radii of curvature), because n appears in the numerator. The optical power of the diamond lens is therefore greater than that of the glass lens.

2. $P_1 = 2.5\ \delta$ $P_2 = 4.0\ \delta$

   The total optical power of the two lenses side by side is:

   $P_T = P_1 + P_2 = 2.5\ \delta + 4.0\ \delta = 6.5\ \delta$

   The focal length of the system formed by the combination of the two lenses is:

$$f_T = \frac{1}{P_T} = \frac{1}{6.5\ \delta} = \frac{1}{6.5\ m^{-1}} = 0.15\ m = 15\ cm$$

3. $f_1 = 10.0\ cm = 0.100\ m$ $f_2 = -15.0\ cm = -0.150\ m$

$$P_1 = \frac{1}{f_1} = \frac{1}{0.100\ m} = 10.0\ m^{-1} = 10.0\ \delta$$

$$P_2 = \frac{1}{f_2} = \frac{1}{-0.150\ m} = -6.67\ m^{-1} = -6.67\ \delta$$

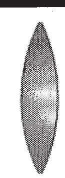
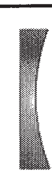
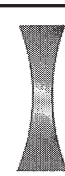
   Considering that the lenses are side by side, the total optical power of the system is equal to:

   $P_T = P_1 + P_2 = 10.0\ \delta + (-6.67\ \delta) = 3.3\ \delta$

   and the focal length of the combination of the two lenses is:

$$f_T = \frac{1}{P_T} = \frac{1}{3.3\ \delta} = \frac{1}{3.3\ m^{-1}} = 0.30\ m = 30\ cm$$

4.

Type of lens	Shape	R_1	R_2
Biconvex		Positive (+)	Negative (-)
Planoconcave		Infinite	Positive (+)
Biconcave		Negative (-)	Positive (+)

5. $f = +0.35\ m$ $P = ?$

$$P = \frac{1}{f} = \frac{1}{0.35\ m} = 2.9\ m^{-1} = 2.9\ \delta$$

6. $P = 3.25\ \delta \qquad f = ?$

$$P = \frac{1}{f} \Rightarrow f = \frac{1}{P} = \frac{1}{3.25\ \delta} = \frac{1}{3.25\ \text{m}^{-1}} = 0.308\ \text{m}$$

7. $P = -5.5\ \delta$

a) $$P = \frac{1}{f} \Rightarrow f = \frac{1}{P} = \frac{1}{-5.5\ \delta} = \frac{1}{-5.5\ \text{m}^{-1}} = -0.18\ \text{m}$$

b) Because $f < 0$, the lens is diverging according to the sign convention.

8. $f = -20.0\ \text{cm} = -0.200\ \text{m} \qquad P = ?$

$$P = \frac{1}{f} = \frac{1}{-0.20\ \text{m}} = -5.0\ \text{m}^{-1} = -5.00\ \delta$$

9. $C = -2.5\ \delta \qquad f = ?$

$$P = \frac{1}{f} \Rightarrow f = \frac{1}{P} = \frac{1}{-2.5\ \delta} = \frac{1}{-2.5\ \text{m}^{-1}} = -0.40\ \text{m}$$

10. $f_1 = 10.0\ \text{cm} = 0.100\ \text{m} \qquad f_2 = 25.0\ \text{cm} = 0.250\ \text{m}$

$$P_1 = \frac{1}{f_1} = \frac{1}{0.100\ \text{m}} = 10.0\ \text{m}^{-1} = 10.0\ \delta$$

$$P_2 = \frac{1}{f_2} = \frac{1}{0.250\ \text{m}} = 4.00\ \text{m}^{-1} = 4.00\ \delta$$

Because the lenses are side by side, the total optical power of the system is equal to:

$P_T = P_1 - P_2 = 10.0\ \delta + 4.00\ \delta = 14.0\ \delta$

and the focal length of the combination of the two lenses is:

$$f_T = \frac{1}{P_T} = \frac{1}{14.0\ \delta} = \frac{1}{14.0\ \text{m}^{-1}} = 0.0714\ \text{m} = 7.14\ \text{cm}$$

11. $P_1 = 2.5\ \delta \qquad P_T = 4.0\ \delta \qquad f_2 = ?$

$P_T = P_1 + P_2$

$P_2 = P_T + P_1 = 4.0\ \delta - 2.5\ \delta = 1.5\ \delta$

$$P_2 = \frac{1}{f_2} \Rightarrow f_2 = \frac{1}{P_2} = \frac{1}{1.5\ \delta} = \frac{1}{1.5\ \text{m}^{-1}} = 0.67\ \text{m}$$

Because $f_2 > 0$, the second lens is converging according to the sign convention.

12. Biconcave lens:

$|R_1| = 12\ \text{cm} \quad |R_2| = 7\ \text{cm} \quad n = 1.52$ (crown glass)

The signs for R_1 and R_2 must be determined by taking the sign convention into account.

Because the lens is biconcave, the centre of curvature of the first surface is on the side of the incident rays, therefore $R_1 = -12\ \text{cm} = -0.12\ \text{m}$.

The centre of curvature of the second surface is on the side of the emerging rays, therefore $R_2 = +7\ \text{cm} = 0.07\ \text{m}$.

$$P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$P = (1.52 - 1)\left(\frac{1}{-0.12\ \text{m}} - \frac{1}{0.07\ \text{m}}\right) = -12\ \text{m}^{-1} = -12\ \delta$$

If we consider that the first surface is the one with the 7-cm radius, we obtain

$R_1 = -0.07\ \text{m}$ and $R_2 = 0.12\ \text{m}$, resulting in the same optical power:

$$P = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$P = (1.52 - 1)\left(\frac{1}{-0.07\ \text{m}} - \frac{1}{0.12\ \text{m}}\right) = -12\ \text{m}^{-1} = -12\ \delta$$

Whether the lens is installed in one way or another does not change its effect on the light rays.

13. Biconvex lens: $n = 1.50 \qquad f = 20\ \text{cm}$

The statement "one radius of curvature (R) that is double that of the other" must be translated into algebraic language:

$|R_1| = 2|R_2|$

We know that: $P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

The two equations above constitute a system of two equations with two unknowns, R_1 and R_2. Before solving them, the absolute values in the first equation must be eliminated. According to the sign convention, for a biconvex lens, $R_1 > 0$ and $R_2 < 0$. Therefore, $R_1 = -2R_2$.

The R_1 expression can now be inserted in the lens-maker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\left(\frac{1}{-2R_2} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n-1)\left(\frac{-1}{2R_2} - \frac{2}{2R_2}\right) = (n-1)\left(\frac{-3}{2R_2}\right)$$

Since $\frac{1}{f} = (n-1)\left(\frac{-3}{2R_2}\right)$, by taking the inverse of both sides, we obtain

$$f = \frac{2R_2}{-3(n-1)}$$

where: $R_2 = \dfrac{-3(n-1)f}{2} = \dfrac{-3(1.50-1)(20\text{ cm})}{2} = -15\text{ cm}$

Since $R_1 = -2R_2$, we obtain
$R_1 = -2 \times (-15\text{ cm}) = 30\text{ cm}$.

If we had written $|R_2| = 2|R_1|$ at the onset instead of $|R_1| = 2|R_2|$, we would have obtained $R_1 = +15$ cm and $R_2 = -30$ cm, which corresponds to the same lens, but inverted.

14. If the lenses have the same shape, it is because R_1 and R_2 have the same values for the two lenses.

Since $P = \dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$,

the only factor that makes the focal length vary is the index of refraction.

$n_{diamond} = 2.42 > n_{crown} = 1.52$, therefore $\left(\dfrac{1}{f}\right)_{diamond} > \left(\dfrac{1}{f}\right)_{crown}$

and therefore $f_{diamond} < f_{crown}$.

It is the crown glass lens that has the longer focal length.

15. $f_1 = 12\text{ cm} = 0.12\text{ m}$ $\quad f_2 = 20\text{ cm} = 0.20\text{ m}$

$$P_1 = \frac{1}{f_1} = \frac{1}{0.12\text{ m}} = 8.3\text{ m}^{-1} = 8.3\ \delta$$

$$P_2 = \frac{1}{f_2} = \frac{1}{0.20\text{ m}} = 5.0\text{ m}^{-1} = 5.0\ \delta$$

Because the lenses are placed side by side, the total optical power of the system is equal to:

$$P_T = P_1 + P_2 = 8.3\ \delta + 5.0\ \delta = 13.3\ \delta$$

and the focal length of the combination of two lenses is:

$$f_T = \frac{1}{P_T} = \frac{1}{13.3\ \delta} = \frac{1}{13.3\text{ m}^{-1}} = 0.075\text{ m} = 7.5\text{ cm}$$

16. $P_1 = +4\ \delta$ $\quad f_2 = -7\text{ cm} = -0.07\text{ m}$

a) $P_2 = \dfrac{1}{f_2} = \dfrac{1}{-0.07\text{ m}} = -14\ \delta$

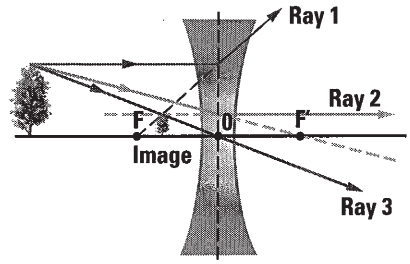
The total optical power of the two lenses combined is: $P_T = P_1 + P_2 = 4\ \delta + (-14\ \delta) = -10\ \delta$

b) This system is diverging, because $P_T < 0$ and $f_T < 0$.

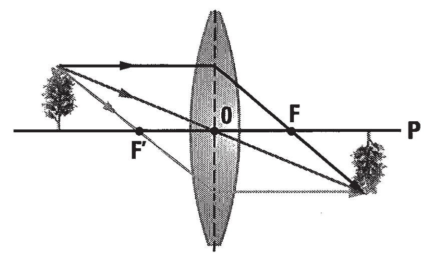
Section 4.4
Images formed by lenses

Textbook, p. 122

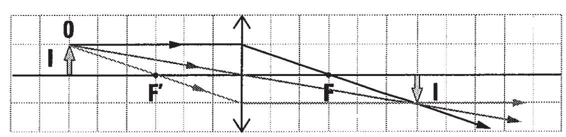
1. The emerging rays are diverging: the image is therefore virtual. The image point corresponding to the object point (top of the tree) is at the meeting point of the extensions (dotted line) of emerging rays.



2. The emerging rays are converting: the image is therefore real. The image point corresponding to the object point (top of the tree) is at the meeting point of the emerging rays.



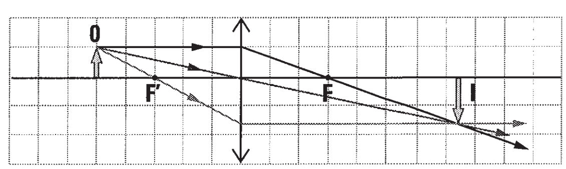
3. *a)*



Scale: 1 square = 5 cm

The image is real (the rays converge after the lens), inverted and its height is similar to that of the object.

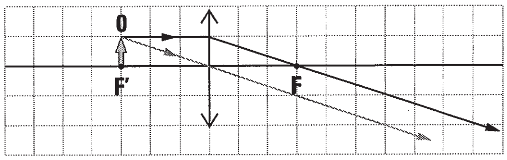
b)



Scale: 1 square = 5 cm

The image is real, inverted and its height is greater (approximately 50% greater) than that of the object.

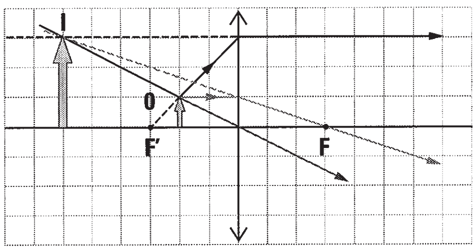
c)



Scale: 1 square = 5 cm

The two principal rays drawn, emerge parallel to the lens and do not meet except at infinity. There is no image or, equivalently, the image is at infinity.

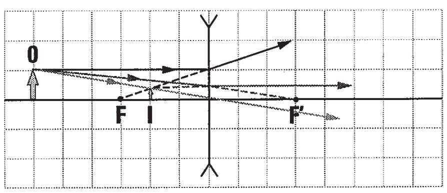
d)



Scale: 1 square = 5 cm

The image is virtual (the rays diverge after having passed through the lens), upright and its height is approximately triple that of the object.

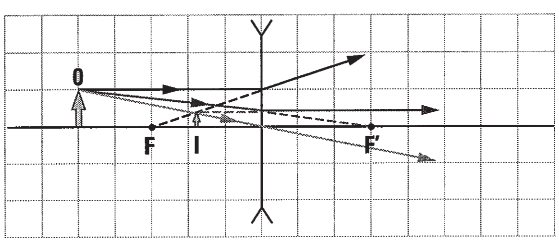
4. *a)*



Scale: 1 square = 5 cm

The image is virtual (the rays diverge after having passed through the lens), upright and smaller than the object.

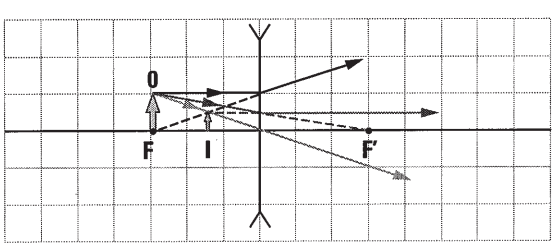
b)



Scale: 1 square = 5 cm

The image is virtual (the rays diverge after having passed through the lens), upright and smaller than the object.

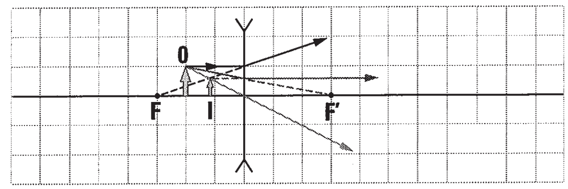
c)



Scale: 1 square = 5 cm

The image is virtual (the rays diverge after having passed through the lens), upright and smaller than the object.

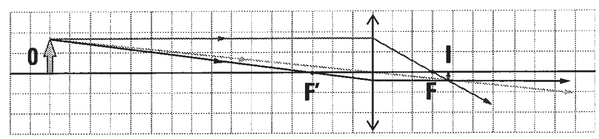
d)



Scale: 1 square = 5 cm

The image is virtual (the rays diverge after having passed through the lens), upright and smaller than the object.

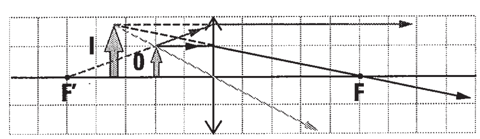
5. In the following diagram, each square corresponds to 5 cm. The focal points are therefore three squares away from the optical centre of the lens.



Scale: 1 square = 5 cm

According to the diagram, the image (real and inverted) is approximately 3.8 squares away from the optical centre, which corresponds to $d_i \cong 19$ cm. The height of the image is equal to approximately 0.4 squares; therefore, $h_i \cong -2$ cm.

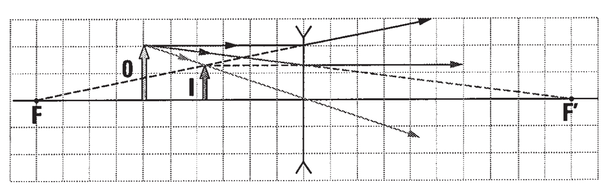
6. In the following diagram, each square corresponds to 5 cm. The focal points are therefore five squares away from the optical centre of the lens.



Scale: 1 square = 5 cm

According to the diagram, the image (virtual and upright) is approximately 3.4 squares away from the optical centre, which corresponds to $d_i \cong -17$ cm. The height of the image is equal to approximately 1.7 squares; therefore, $h_i \cong +8.5$ cm.

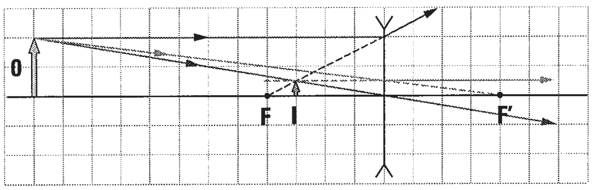
7. In the following diagram, each square corresponds to 2.5 cm. The focal points are therefore 10 squares away from the optical centre of the lens.



Scale: 1 square = 2.5 cm

According to the diagram, the image (virtual and upright) is approximately 3.7 squares away from the optical centre, which corresponds to $d_i \cong$ -9 cm. The height of the image is equal to approximately 1.2 squares; therefore, $h_i \cong +3$ cm.

8. In the following diagram, each square corresponds to 5 cm. The focal points are therefore four squares away from the optical centre of the lens.



Scale: 1 square = 5 cm

According to the diagram, the image (virtual and upright) is approximately three squares away from the optical centre, which corresponds to $d_i \cong$ -15 cm. The height of the image is equal to approximately 0.5 squares; therefore, $h_i \cong +2.5$ cm.

9. For question 6: $d_o = 10$ cm $\quad h_o = 5$ cm $\quad f = 25$ cm

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{1}{25\text{ cm}} - \frac{1}{10\text{ cm}} = \frac{2}{50\text{ cm}} - \frac{5}{50\text{ cm}} = \frac{-3}{50\text{ cm}}$$

$$d_i = \frac{-50\text{ cm}}{3} = -17\text{ cm}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$h_i = -\frac{d_i h_o}{d_o} = -\frac{(-17\text{ cm}) \times 5\text{ cm}}{10\text{ cm}} = +8.5\text{ cm}$$

For question 7: $d_o = 15$ cm $\quad h_o = 5$ cm
$f = -25$ cm

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{1}{-25\text{ cm}} - \frac{1}{15\text{ cm}} = \frac{-3}{75\text{ cm}} - \frac{5}{75\text{ cm}} = \frac{-8}{75\text{ cm}}$$

$$d_i = \frac{-75\text{ cm}}{8} = -9.4\text{ cm}$$

$$g = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$h_i = -\frac{d_i h_o}{d_o} = -\frac{(-9.4\text{ cm}) \times 5\text{ cm}}{15\text{ cm}} = +3.1\text{ cm}$$

For question 8: $d_o = 60$ cm $\quad h_o = 10$ cm
$f = -20$ cm

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{1}{-20\text{ cm}} - \frac{1}{60\text{ cm}} = \frac{-3}{60\text{ cm}} - \frac{1}{60\text{ cm}} = \frac{-4}{60\text{ cm}}$$

$$d_i = \frac{-60\text{ cm}}{4} = -15\text{ cm}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$h_i = -\frac{d_i h_o}{d_o} = -\frac{(-15\text{ cm}) \times 10\text{ cm}}{60\text{ cm}} = +2.5\text{ cm}$$

For the three cases, the results obtained through calculations are consistent with those obtained by tracing the principal rays.

10. $f = +20$ cm (converging lens) $\quad |M| = 4 \quad d_o = ?$
$d_i = ?$

There are two answers to this question, depending on the sign for the magnification.

1° $M = +4$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = +4 \Rightarrow d_i = -4d_o$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{+20\text{ cm}}$$

The two equations above constitute a system of two equations with two unknowns, d_o and d_i, that must be solved. For example, in the second equation, we can replace the d_i expression provided by the first equation:

$$\frac{1}{d_o} + \frac{1}{-4d_o} = \frac{4}{4d_o} - \frac{1}{4d_o} = \frac{3}{4d_o} = \frac{1}{20\text{ cm}}$$

$4d_o = 60$ cm

$\Rightarrow d_o = 15$ cm

$d_i = -4d_o = -4 \times 15$ cm

$\Rightarrow d_i = -60$ cm

The image is virtual, therefore, it is on the same side of the lens as the object. The distance between the object and the image is:

$\Delta d = |d_i| - d_o = 60\text{ cm} - 15\text{ cm} = 45\text{ cm}$

2° $M = -4$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = -4 \Rightarrow d_i = 4d_o$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{+20\text{ cm}}$$

The two equations above constitute a system of two equations with two unknowns, d_o and d_i, that must be solved. For example, in the second equation, we can replace the d_i expression provided by the first equation:

$$\frac{1}{d_o} + \frac{1}{4d_o} = \frac{4}{4d_o} + \frac{1}{4d_o} = \frac{5}{4d_o} = \frac{1}{20\text{ cm}}$$

$\Rightarrow 4d_o = 100\text{ cm} \Rightarrow d_o = 25\text{ cm}$

$d_i = 4d_o = 4 \times 25\text{ cm}$

$d_i = 100\text{ cm}$

The image is real and is on the other side of the lens in relation to the object. The distance between the object and the image is:

$d_o + d_i = 25\text{ cm} + 100\text{ cm} = 125\text{ cm}$

11.

Lens	*Converging*	*Diverging*	Converging	Diverging	Converging
f (cm)	*20*	*-20*	10	-30	6.7
d_o (cm)	*25*	*25*	*20*	*15*	20
d_i (cm)	100 cm	-11 cm	*20*	*-10*	*10*
M	-4.0	+0.44	*-1*	+0.67	*-0.5*
Real/virtual image	*Real*	*Virtual*	Real	Virtual	*Real*
Orientation	Inverted	Upright	Inverted	Upright	*Inverted*

1st lens: converging

$f = 20\text{ cm} \quad d_o = 25\text{ cm} \quad d_i = ? \quad g = ?$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} - \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{20\text{ cm}} - \frac{1}{25\text{ cm}}$$

$$\frac{1}{d_i} = \frac{5}{100\text{ cm}} - \frac{4}{100\text{ cm}} = \frac{1}{100\text{ cm}} \qquad d_i = 100\text{ cm}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{100\text{ cm}}{25\text{ cm}} = -4.0$$

Because $M < 0$, the image is inverted.

2nd lens: diverging

$f = -20\text{ cm} \quad d_o = 25\text{ cm} \quad d_i = ? \quad M = ?$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-20\text{ cm}} - \frac{1}{25\text{ cm}}$$

$$\frac{1}{d_i} = \frac{-5}{100\text{ cm}} - \frac{4}{100\text{ cm}} = \frac{-9}{100\text{ cm}} \Rightarrow d_i = -11\text{ cm}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{-11\text{ cm}}{25\text{ cm}} = +0.44$$

Because $M < 0$, the image is upright.

3rd lens

$d_o = 20\text{ cm} \quad d_i = 20\text{ cm} \quad M = -1 \quad f = ?$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{20\text{ cm}} + \frac{1}{20\text{ cm}} = \frac{2}{20\text{ cm}} = \frac{1}{10\text{ cm}}$$

$\Rightarrow f = 10\text{ cm}$

Because $f > 0$, the lens is converging.

Because $d_i > 0$, the image is real.

Because $M < 0$, the image is inverted.

4th lens

$d_o = 15\text{ cm} \quad d_i = -10\text{ cm} \quad M = ? \quad f = ?$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{15\text{ cm}} + \frac{1}{-10\text{ cm}} = \frac{2}{30\text{ cm}} - \frac{3}{30\text{ cm}}$$

$$\frac{1}{f} = \frac{-1}{30\text{ cm}} \Rightarrow f = -30\text{ cm}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{-10\text{ cm}}{15\text{ cm}} = +0.67$$

Because $f < 0$, the lens is diverging.

Because $d_i < 0$, the image is virtual.

Because $M > 0$, the image is upright.

5th lens

$d_o = ? \quad d_i = 10\text{ cm} \quad g = -0.5 \quad f = ?$

$$M = -\frac{d_i}{d_o} = -\frac{10\text{ cm}}{d_o} = -0.5 \Rightarrow 0.5d_o = 10\text{ cm}$$

$d_o = 20\text{ cm}$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{20\text{ cm}} + \frac{1}{10\text{ cm}} = \frac{1}{20\text{ cm}} + \frac{2}{20\text{ cm}}$$

$$\frac{1}{f} = \frac{3}{20\text{ cm}} \Rightarrow f = \frac{20\text{ cm}}{3} = 6.7\text{ cm}$$

Because $f > 0$, the lens is converging.

Because $d_i > 0$, the image is real.

Because $g < 0$, the image is inverted.

12. $f = +10$ cm $\quad M = +2 \quad d_o = ? \quad d_i = ?$

a, b) $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = +2 \Rightarrow d_i = -2d_o$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{+10 \text{ cm}}$$

The two equations above constitute a system of two equations with two unknowns, d_o and d_i, that must be solved. For example, in the first equation, we can replace the d_i expression provided by the first equation:

$$\frac{1}{d_o} + \frac{1}{-2d_o} = \frac{1}{d_o} \Rightarrow \frac{1}{2d_o} = \frac{2}{2d_o} - \frac{1}{2d_o} = \frac{1}{2d_c}$$

$$\frac{1}{2d_o} = \frac{1}{10 \text{ cm}} \Rightarrow 2d_o = 10 \text{ cm} \Rightarrow d_o = 5 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{10 \text{ cm}} - \frac{1}{5 \text{ cm}}$$

$$\frac{1}{d_i} = \frac{1}{10 \text{ cm}} - \frac{2}{10 \text{ cm}} = \frac{-1}{10 \text{ cm}}$$

$\Rightarrow d_i = -10$ cm

(We confirm that $M = -\frac{d_i}{d_o} = -\frac{-10 \text{ cm}}{5 \text{ cm}} = +2$)

c) Because $f > 0$, according to sign convention, the lens is converging.

13. $f = -20$ cm $\quad M = +0.5 \quad d_o = ? \quad d_i = ?$

a, b) $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = +0.5 \Rightarrow d_i = -0.5d_o$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{-20 \text{ cm}}$$

The two equations above constitute a system of two equations with two unknowns, d_o and d_i, that must be solved. For example, in the second equation, we can replace the d_i expression provided by the first equation:

$$\frac{1}{d_o} + \frac{1}{-0.5d_o} = \frac{1}{d_o} - \frac{2}{d_o} = \frac{-1}{d_o} = \frac{1}{-20 \text{ cm}}$$

$\Rightarrow d_o = 20$ cm

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{1}{-20 \text{ cm}} - \frac{1}{20 \text{ cm}} = \frac{-2}{20 \text{ cm}}$$

$\Rightarrow d_i = -10$ cm

(We confirm that $M = -\frac{d_i}{d_o} = -\frac{-10 \text{ cm}}{20 \text{ cm}} = +0.5$)

c) Because $f < 0$, according to the sign convention, the lens is diverging.

14. $f = +8$ cm

If the candle is 36 cm from the screen and a clear (real) image is formed on the screen, then $d_o + d_i = 36$ cm.

However:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{8 \text{ cm}}$$

The two equations above constitute a system of two equations with two unknowns, d_o and d_i, that must be solved. For example, in the first equation, we can isolate d_i and replace the expression obtained in the second equation:

$d_i = 36 \text{ cm} - d_o$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{36 \text{ cm} - d_o}$$

$$= \frac{36 \text{ cm} - d_o}{d_o(36 \text{ cm} - d_o)} + \frac{d_o}{d_o(36 \text{ cm} - d_o)}$$

$$= \frac{36 \text{ cm}}{d_o(36 \text{ cm} - d_o)} = \frac{1}{8 \text{ cm}}$$

$d_o\,(36 \text{ cm} - d_o) = 36 \text{ cm} \times 8 \text{ cm}$

$-d_o^2 + (36 \text{ cm})d_o = 288 \text{ cm}^2$

or $d_o^2 - (36 \text{ cm})d_o + 288 \text{ cm}^2 = 0$

this is a second-degree equation of the form $ax^2 + bx + c = 0$ with $a = 1$, $b = -36$ and $c = 288$. The solutions are:

$$d_o = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$d_o = \frac{36 \pm \sqrt{36^2 - 4 \times 1 \times 288}}{2} \text{ cm}$$

$d_o = 24$ cm or 12 cm

A clear image is obtained for $d_o = 24$ cm (then $d_i = 12$ cm), or for $d_o = 12$ cm (then $d_i = 24$ cm).

Section 4.5
Optical aberrations of lenses

Textbook, p. 124

1. A chromatic aberration is an optical defect caused by the fact that incident rays of different wavelengths are not refracted in the same way by a lens. This phenomenon is due to the variation in the index of refraction according to the light's wavelength. This difference in refraction has the effect of focusing parallel incident rays of different wavelengths on different points of the principal axis (P).

   A chromatic aberration can be corrected by using an achromatic doublet.

2. *a)* In accordance with the lens-maker's equation, the focal length (f) of a lens depends on the index of refraction (n) of the material used to make it. Because the index of refraction depends on the light's wavelength (λ) the focal length will, therefore, depend on the colour of the beam that passes through the lens.

   b) We must consider:

   - the lens-maker's equation:

     $$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

     (*see Section 4.3 on page 110 in the textbook*)

   - the way the index of refraction varies with the wavelength:

     $$n(\lambda) = A + \frac{B}{\lambda^2}$$

     (*see Section 3.2 on page 81 in the textbook*)

   - the fact that the wavelength of the red light (λ_R) is greater than the wavelength of the green light (λ_g).

   Because $\lambda_R > \lambda_g$, according to $n(\lambda) = A + \frac{B}{\lambda^2}$, we obtain $n_R < n_g$.

   According to $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$,

   we obtain $\left(\frac{1}{f_R}\right) < \left(\frac{1}{f_G}\right)$, and therefore $f_R > f_g$.

   Therefore, we obtain the shorter focal length with the green light.

3. Spherical aberration is caused by the spherical shape of the surfaces that define the lens. Because of this lens' particular geometrical shape, it does not focus the parallel rays that pass near the principal axis (P) and those that pass at the periphery of the lens on the same point

Chapter 4
Lenses

Textbook, p. 130

1. Unlike with lenses, light does not pass through mirrors. Mirrors reflect light (the phenomenon of reflection) that therefore, travels in the same medium. In the case of lenses, light passes through them, thereby changing their medium. Passing through a lens prompts the phenomenon of refraction, which depends on the light's wavelength (λ). The refracted light's trajectory depends on the index of refraction (n) of the material with which the lens is made. Because this index itself depends on the light's wavelength, the refracted light's trajectory will also depend on it. It is this difference in trajectory that causes chromatic aberrations.

2. Planoconvex lens: $f = 10$ cm
   Material: glass $\Rightarrow n = 1.50$

   $$P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

   For a planoconvex lens, one of the surfaces, e.g. the second one, has an infinite radius of curvature: $R_2 = \infty$; therefore, $\frac{1}{R_2} = 0$.

   The first surface is convex, $R_1 > 0$ according to the sign convention. Therefore,

   $$\frac{1}{f} = \frac{(n-1)}{R_1}$$

   $$R_1 = (n-1)\,f = (1.50 - 1) \times 10 \text{ cm} = 5 \text{ cm}$$

   If the flat surface is the first one: $R_1 = \infty$; therefore,

   $$\frac{1}{R_1} = 0.$$

   The second surface is convex, $R_2 < 0$ according to the sign convention. We obtain

$$\frac{1}{f} = (n-1)\left(\frac{-1}{R_2}\right) = \frac{(n-1)}{|R_2|}$$

and, therefore $|R_2| = 5$ cm as well.

3. $f_1 = 20$ cm $f_2 = -10$ cm $f_T = 40$ cm $f_3 = ?$

The three lenses are placed side by side; therefore $P_T = P_1 + P_2 + P_3$. If f_1, f_2 and f_T are known, we can determine P_1, P_2 and P_T, and then P_3 and f_3.

$$P_1 = \frac{1}{f_1} = \frac{1}{20\text{ cm}} = \frac{1}{0.20\text{ m}} = 5.0\text{ m}^{-1} = 5.0\ \delta$$

$$P_2 = \frac{1}{f_2} = \frac{1}{-10\text{ cm}} = \frac{1}{-0.10\text{ m}} = -10\text{ m}^{-1} = -10\ \delta$$

$$P_T = \frac{1}{f_T} = \frac{1}{40\text{ cm}} = \frac{1}{0.40\text{ m}} = 2.5\text{ m}^{-1} = 2.5\ \delta$$

$$P_T = P_1 + P_2 + P_3 \qquad P_3 = P_T - P_1 - P_2$$

$$P_3 = 2.5\ \delta - 5.0\ \delta - (-10\ \delta) = 7.5\ \delta$$

$$f_3 = \frac{1}{P_3} = \frac{1}{7.5\ \delta} = \frac{1}{7.5\text{ m}^{-1}} = 0.13\text{ m} = 13\text{ cm}$$

The third lens has a focal length of 13 cm. Because $f_3 > 0$, the lens is converging.

4. Planoconvex lens: $R_1 = \infty$ $R_2 < 0$

$$P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\left(0 - \frac{1}{R_2}\right)$$

$$P = \frac{-(n-1)}{R_2}$$

Because $R_2 < 0$, P and f are positive. Therefore, according to the sign convention, the lens is converging.

We can also consider that $R_2 = \infty$; then, $R_1 > 0$.

In this case, we obtain $\frac{1}{f} = \frac{(n-1)}{R_1}$,

which amounts to the same (inverting the lens does not change the focal length).

5. Planoconvex lens: $R_1 = \infty$ $R_2 = -25$ cm

(We could say that $R_1 = +25$ cm and $R_2 = \infty$ and we would obtain the same results; inverting the lens does not change its focal length.)

$n_b = 1.523$ $n_r = 1.514$

a) As covered in question 4, for a planoconvex lens,

$$\frac{1}{f} = \frac{-(n-1)}{R_2}, \text{ therefore } f = \frac{-R_2}{(n-1)}.$$

For the blue light:

$$f_b = \frac{-R_2}{(n_b - 1)} = \frac{-(-25\text{ cm})}{(1.523 - 1)} = 47.80\text{ cm}$$

For the red light:

$$f_r = \frac{-R_2}{(n_r - 1)} = \frac{-(-25\text{ cm})}{(1.514 - 1)} = 48.64\text{ cm}$$

The distance that separates the two focal points is $d = f_r - f_b = 48.64\text{ cm} - 47.80\text{ cm} = 0.84$ cm.

b) This phenomenon is called chromatic aberration.

6. Diverging meniscus: $R_1 = -10$ cm $R_2 = -22$ cm

(We could say that $R_1 = +22$ cm and $R_2 = +10$ cm and we would obtain the same results; inverting the lens does not change its focal length.)

Because the lens is diverging, $f = -35$ cm (according to the sign convention).

By using the lens-maker's equation, we can isolate the lens' index of refraction:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow n - 1 = \frac{\frac{1}{f}}{\frac{1}{R_1} - \frac{1}{R_2}}$$

$$n - 1 = \frac{\frac{1}{-35\text{ cm}}}{\frac{1}{-10\text{ cm}} - \frac{1}{-22\text{ cm}}} = \frac{\frac{-1}{35\text{ cm}}}{\frac{-22}{220\text{ cm}} + \frac{10}{220\text{ cm}}}$$

$$= \frac{\frac{-1}{35\text{ cm}}}{\frac{-12}{220\text{ cm}}} = \frac{-1 \times 220\text{ cm}}{-12 \times 35\text{ cm}}$$

$n - 1 = 0.52 \Rightarrow n = 1.52$

The index of refraction obtained is that of a material with the same index of refraction as that of crown glass (*see Table 1 on page 79 in the textbook*).

7. Lens 1: $R_{11} = \infty$ $R_{21} = -15$ cm (light is presumed to come from the left)

Lens 2: $R_{12} = -15\text{ cm} = R$ $R_{22} = \infty$

a) Parallel incident rays will emerge parallel to the system, because the two vertical interfaces are parallel (the optical system acts like glass). We can say that the rays are focused at infinity and write $f = \infty$, therefore

$$P = \frac{1}{f} = \frac{1}{\infty} = 0.$$

We can also proceed more formally:

Because $R_{12} = R_{21}$, we consider that $R_{12} = R_{21} = R$ where $R < 0$.

Because the two lenses are side by side, $P_T = P_1 + P_2$. We calculate P_1 and P_2:

$$P_1 = (n-1)\left(\frac{1}{R_{11}} - \frac{1}{R_{21}}\right) = (n-1)\left(\frac{1}{\infty} - \frac{1}{R}\right)$$

$$P_1 = -\frac{(n-1)}{R}$$

$$P_2 = (n-1)\left(\frac{1}{R_{12}} - \frac{1}{R_{22}}\right)$$

$$P_2 = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{(n-1)}{R}$$

$$P_T = P_1 + P_2 = -\frac{(n-1)}{R} + \frac{(n-1)}{R} = 0$$

b) $n_1 = 1.50$ and $n_2 = 1.66$ (*see Table 1 on page 79 in the textbook*)

$$P_1 = (n-1)\left(\frac{1}{R_{11}} - \frac{1}{R_{21}}\right)$$

$$P_1 = (1.50-1)\left(\frac{1}{\infty} - \frac{1}{-0.15\text{ m}}\right) = \frac{0.50}{0.15\text{ m}} = 3.3\text{ m}^{-1}$$

$= 3.3\ \delta$

$$P_2 = (n-1)\left(\frac{1}{R_{12}} - \frac{1}{R_{22}}\right)$$

$$P_2 = (1.66-1)\left(\frac{1}{-0.15\text{ m}} - \frac{1}{\infty}\right) = \frac{-0.66}{0.15\text{ m}} = -4.4\text{ m}^{-1}$$

$= -4.4\ \delta$

$$P_T = P_1 + P_2 = 3.3\ \delta - 4.4\ \delta = -1.1\ \delta$$

Since $P < 0$, then $f < 0$ and, according to the sign convention, the optical system is diverging.

8. $d_o = 10$ cm $h_o = 0.5$ cm $P = 5\ \delta$ $d_i = ?$ $M = ?$

a) $P = \frac{1}{f}$

$$f = \frac{1}{P} = \frac{1}{5\ \delta} = \frac{1}{5\text{ m}^{-1}} = 0.20\text{ m} = 20\text{ cm}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{1}{20\text{ cm}} - \frac{1}{10\text{ cm}} = \frac{1}{20\text{ cm}} - \frac{2}{20\text{ cm}} = \frac{-1}{20\text{ cm}}$$

$d_i = -20$ cm

b) $M = -\frac{d_i}{d_o} = -\frac{-20\text{ cm}}{10\text{ cm}} = +2.0$

c) Because $f > 0$, the lens is converging according to the sign convention.

9. $|M| = 0.5$ $M = -5\ \delta$ $d_o = ?$ $d_i = ?$

Because the lens is converging ($P < 0$), the image is upright (*see Table 3 on page 114 in the textbook*) and $M = +0.5$.

a) $P = \frac{1}{f} \Rightarrow f = \frac{1}{P} = \frac{1}{-5\ \delta} = \frac{1}{-5\text{ m}^{-1}}$

$f = -0.20\text{ m} = -20\text{ cm}$

$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = -\frac{1}{20\text{ cm}}$ and $M = -\frac{d_i}{d_o} = 0.5$

These two equations form a two-equation system with two unknowns, d_o and d_i. We isolate d_i in the second equation: $d_i = -0.5\ d_o$ and we insert the expression obtained in the first equation:

$$\frac{1}{d_o} + \frac{1}{-0.5d_o} = \frac{1}{d_o} - \frac{2}{d_o} = \frac{-1}{d_o} = -\frac{1}{20\text{ cm}}$$

$\Rightarrow d_o = 20$ cm

b) $d_i = -0.5d_o = -0.5 \times (20\text{ cm})$

$d_i = -10$ cm

c) Because $f < 0$, the lens is diverging according to the sign convention.

10. Biconvex glass lens: $n = 1.50$

$|R_1| = 12$ cm $|R_2| = 2 \times |R_1| = 24$ cm

Considering the sign convention, $R_1 = +12$ cm and $R_2 = -24$ cm.

(We could say that $R_1 = +24$ cm and $R_2 = -12$ cm and we would obtain the same results; inverting the lens does not change its focal length.)

a) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f} = (1.50-1)\left(\frac{1}{12 \text{ cm}} - \frac{1}{-24 \text{ cm}}\right)$

$\frac{1}{f} = 0.50 \times \left(\frac{2}{24 \text{ cm}} + \frac{1}{24 \text{ cm}}\right)$

$\frac{1}{f} = 0.50 \times \frac{3}{24 \text{ cm}} = \frac{1}{2} \times \frac{1}{8 \text{ cm}} = \frac{1}{16 \text{ cm}}$

$\Rightarrow f = 16 \text{ cm}$

b) $d_o = 20$ cm $\quad d_i = ?$

$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{16 \text{ cm}} - \frac{1}{20 \text{ cm}}$

$\frac{1}{d_i} = \frac{5}{80 \text{ cm}} - \frac{4}{80 \text{ cm}} = \frac{1}{80 \text{ cm}} \Rightarrow d_i = 80 \text{ cm}$

c) $M = -\frac{d_i}{d_o} = -\frac{80 \text{ cm}}{20 \text{ cm}} = -4$

The image is therefore real ($d_i > 0$) and inverted ($M < 0$).

Chapter 5 Applied Geometric Optics

Textbook, p. 131 to 147

PRACTICE MAKES PERFECT

Section 5.1 The camera

Textbook, p. 133

1. The rays from a point on the object diverge and the objective must make these rays converge on one point on the light-sensitive surface: the objective is therefore converging.

2. The image formed on a camera's light-sensitive surface is real and inverted.

3. The distance of an object being photographed (the object distance d_o) can vary from one photo to another. According to the thin-lens equation, the image distance d_i also varies. If the objective was always in the same place, the image could form in front of or behind the light-sensitive surface, which would result in blurry photos.

   To bring the image into focus, i.e. ensure that the image is formed exactly on the light-sensitive surface and that the picture will be clear, you only need to modify the distance between the lens and the light-sensitive surface. Most cameras automatically make this adjustment.

4. According to the thin-lens equation: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

   when $d_o \to \infty$, $\frac{1}{d_o} \to 0$.

   Then: $\frac{1}{f} = 0 + \frac{1}{d_i} = \frac{1}{d_i} \Rightarrow f = d_i$

   Therefore, when the object being photographed is at an infinite distance (or very far away) from the lens, the distance between the objective and the light-sensitive surface is equal to the focal length (f).

5. $f = 6.0$ cm

   Because the objective is 7.0 cm from the light-sensitive surface and the image is clear, then $d_i = 7.0$ cm.

   $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i}$

   $\frac{1}{d_o} = \frac{1}{6.0 \text{ cm}} - \frac{1}{7.0 \text{ cm}} = \frac{7}{42 \text{ cm}} - \frac{6}{42 \text{ cm}} = \frac{1}{42 \text{ cm}}$

   $d_o = 42$ cm

6. $f = 6.0$ cm $\quad R_M = 1.74 \times 10^6$ m $\quad d_{EM} = 3.84 \times 10^8$ m

   $d_o = 3.84 \times 10^8$ m $\quad h_o = 2 \times R_M = 3.48 \times 10^6$ m

   $h_i = ?$

Because $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$, to find h_i, we must first find d_i:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{6.0\text{ cm}} - \frac{1}{3.84 \times 10^8\text{ m}}$$

The object is so far away that the denominator of the subtraction's second term is very large, which makes the second term negligible:

$$\frac{1}{d_i} \approx \frac{1}{f} \Rightarrow d_i = f = 6.0\text{ cm}$$

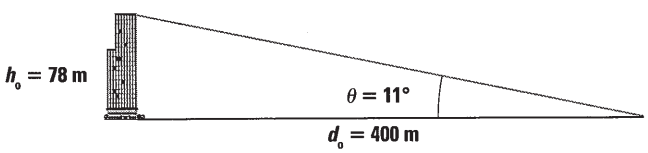
$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$h_i = -\frac{h_o d_i}{d_o} = -\frac{3.48 \times 10^6\text{ m} \times 6.0\text{ cm}}{3.84 \times 10^8\text{ m}} = -0.054\text{ cm}$$

The size of the Moon's image on the light-sensitive surface is 0.54 mm.

7. Object: building $d_o = 400$ m $h_o = 78$ m
Converging lens: $f = 5$ cm

a) It is presumed that the viewfinder does not deviate the rays.



$$\tan\theta = \frac{h_o}{d_o} = \frac{78\text{ m}}{400\text{ m}} = 0.20$$

$$\theta = \tan^{-1}(0.20) = 11°$$

The tourist sees the building at an 11° angle.

b) $d_i = ?$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{1}{5\text{ cm}} - \frac{1}{400\text{ m}} = \frac{1}{0.05\text{ m}} - \frac{1}{400\text{ m}}$$

$$\frac{1}{d_i} = \frac{8000}{400\text{ m}} - \frac{1}{400\text{ m}} = \frac{7999}{400\text{ m}}$$

$$d_i = \frac{400\text{ m}}{7999} = 0.05001\text{ m} \approx 5\text{ cm}$$

The image distance is practically equal to the lens' focal length. The image will therefore, form on the film.

c) $h_i = ?$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$h_i = -\frac{h_o d_i}{d_o} = -\frac{78\text{ m} \times 5\text{ cm}}{400\text{ m}} = -0.98\text{ cm}$$

The image is inverted and its size will be equal to 9.8 mm.

Section 5.2 The human eye

Textbook, p. 138

1. $P = 5.2\ \delta$

Because $P > 0$, the lens is converging. The eye is therefore, hyperopic, a defect that requires an increase in optical power.

2. The human eye is made in such a way that the cornea-crystalline lens optical system focuses rays so as to have a clear image on the retina when the eye is in air.

If the eye is in water, the cornea's refraction is much less pronounced, because we have $n_1 = 1.33$ instead of $n_1 = 1.00$. The eye is then no longer able to sufficiently deviate the rays to focus them on the retina and images are blurry. The diving mask or swimming goggles make it possible to interpose a layer of air in front of the eye, and therefore, for the eye to return to its usual situation.

3. A myopic person wears diverging corrective glasses. As indicated in Chapter 4 (*see Table 3 on page 114*), a diverging lens always provides a virtual and upright image that is smaller than the object. It is this image that you see when you look at a person's eyes through their glasses. The eyes of a myopic person, therefore, appear smaller than they really are.

If a person is hyperopic (or presbyopic), they wear converging glasses. The surface of their eyes is between the focal point and the lens. As indicated in Chapter 4 (*see Table 2 on page 114*), in these conditions, a converging lens provides a virtual and upright image that is larger than the object. Seen through glasses, the eyes of the hyperopic person, therefore, appear larger than they really are.

4. When an object is very far away (object at far point), $d_o \to \infty$ and $d_i = f$. The first sentence of the problem, therefore, provides the eye's focal length for an object at far point (FP): $f_{FP} = 2.0$ cm. The eye's refraction is then:

$$P_{FP} = \frac{1}{f_{FP}} = \frac{1}{2.0\,\text{cm}} = \frac{1}{0.020\,\text{m}} = 50\,\text{m}^{-1} = 50\,\delta$$

If you consider the far point (FP) located at 0.40 m for an object located at this distance, the image must still be formed on the retina; therefore,

$d_o = 0.40$ m $\quad d_i = 2.0$ cm $\quad f_{NP} = ?$

$$P_{NP} = \frac{1}{f_{NP}} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.40\,\text{m}} + \frac{1}{0.020\,\text{m}}$$

$$= \frac{1}{0.40\,\text{m}} + \frac{20}{0.40\,\text{m}} = \frac{21}{0.40\,\text{m}} = 53\,\delta$$

The amplitude of accommodation is:

$$\Delta P = P_{NP} - P_{FP} = 53\,\delta - 50\,\delta = 3\,\delta$$

5. *a)* Presbyopia is a visual disorder caused by the aging of the crystalline lens that no longer sufficiently accommodates. For an object close to the eye, the image is formed behind the retina. The far point of a normal eye is approximately 25 cm.

b) Because the image is formed on the retina when the object is at the far point, we have, for an eye without correction:

$d_o = 45$ cm $\quad d_i = 17$ mm $= 1.7$ cm $\quad f_{NP} = ?$

It is therefore possible to calculate the optical power (P_{NP}) of the eye, without correction, for an object at far point:

$$\frac{1}{f_{NP}} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{45\,\text{cm}} + \frac{1}{1.7\,\text{cm}}$$

$$= 0.022\,\text{cm}^{-1} + 0.588\,\text{cm}^{-1} = 0.610\,\text{cm}^{-1}$$

$$f_{NP} = 1.64\,\text{cm}$$

$$P_{NP} = \frac{1}{f_{NP}} = \frac{1}{0.0164\,\text{m}} = 61\,\delta$$

With correction, for an object at 25 cm, the image must also be formed on the retina; therefore, in this case,

$d_o' = 25$ cm $\quad d_i' = 17$ mm $= 1.7$ cm $\quad f_{NP}' = ?$

$$\frac{1}{f_{NP}'} = \frac{1}{d_o'} + \frac{1}{d_i'} = \frac{1}{25\,\text{cm}} + \frac{1}{1.7\,\text{cm}}$$

$$= 0.040\,\text{cm}^{-1} + 0.588\,\text{cm}^{-1} = 0.628\,\text{cm}^{-1}$$

$$f_{NP}' = 1.59\,\text{cm}$$

$$P_{NP}' = \frac{1}{f_{NP}'} = \frac{1}{0.0159\,\text{m}} = 63\,\delta$$

The corrective lens is placed beside the eye, therefore, its optical power (P_L) is obtained by:

$$P_L + P_{NP} = P_{NP}'$$

$$P_l = P_{NP}' - P_{NP} = 63\,\delta - 61\,\delta = 2\,\delta$$

Because $P_L > 0$, the lens is converging.

Section 5.3
The light microscope

Textbook, p. 140

1. The objective and the eyepiece.

2. The lenses used in light microscopes are converging.

3. The image formed by the eyepiece lens of a light microscope is virtual. To observe it, you must look into the eyepiece; the image cannot be projected onto a screen.

4. Consider L_1, the objective, and L_2, the eyepiece.

Image provided by the objective:

$d_{o1} = 2.5$ cm $\quad h_{o1} = 1.0$ cm $\quad f_1 = 2.0$ cm

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \Rightarrow \frac{1}{d_{i1}} = \frac{1}{f_1} - \frac{1}{d_{o1}} = \frac{1}{2.0\,\text{cm}} - \frac{1}{2.5\,\text{cm}}$$

$$\frac{1}{d_{i1}} = \frac{5-4}{10\,\text{cm}} = \frac{1}{10\,\text{cm}} \Rightarrow d_{i1} = 10\,\text{cm}$$

The objective forms a real image 10 cm from the objective.

$$M_1 = \frac{h_{i1}}{h_{o1}} = -\frac{d_{i1}}{d_{o1}}$$

$$h_{i1} = -\frac{h_{o1}d_{i1}}{d_{o1}} = -\frac{1.0\,\text{cm} \times 10\,\text{cm}}{2.5\,\text{cm}} = -4.0\,\text{cm}$$

After having crossed one another where the image is located, the rays continue to travel and reach the eyepiece. For the eyepiece, the object is, by definition, the meeting point of the incident rays on the eyepiece. But this meeting point also corresponds to the meeting point for the rays emerging from the objective, which is the position of the objective's image. In other words, the image provided by the objective becomes the object for the eyepiece.

Because the eyepiece is 12 cm from the objective and the image provided by the objective is 10 cm from the objective (therefore 2 cm from the eyepiece), the object distance for the eyepiece is 2 cm. The object height for the eyepiece is equal to the image height provided by the objective.

Image provided by the eyepiece:

$d_{o2} = 2\text{ cm} \quad h_{o2} = -4.0\text{ cm} \quad f_2 = 2.3\text{ cm}$

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \Rightarrow \frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o2}}$$

$$\frac{1}{d_{i2}} = \frac{1}{2.3\text{ cm}} - \frac{1}{2\text{ cm}} = \frac{2-2.3}{4.6\text{ cm}} = \frac{-0.3}{4.6\text{ cm}}$$

$$\Rightarrow d_{i2} = \frac{-4.6\text{ cm}}{0.3} = -15\text{ cm}$$

The eyepiece forms a virtual image 15 cm from the eyepiece of the following size:

$$M_2 = \frac{h_{i2}}{h_{o2}} = -\frac{d_{i2}}{d_{o2}}$$

$$h_{i2} = -\frac{h_{o2}d_{i2}}{d_{o2}} = -\frac{-4.0\text{ cm} \times (-15\text{ cm})}{2\text{ cm}} = -30\text{ cm}$$

The final image, formed by the eyepiece, is much larger than the object, and it is inverted, as observed when using a microscope.

5. Consider L_1, the objective, and L_2, the eyepiece.

a) The object is the letter:

$d_{o1} = 1.7\text{ cm} \quad h_{o1} = 1.0\text{ mm} \quad f_1 = 1.5\text{ cm}$

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \Rightarrow \frac{1}{d_{i1}} = \frac{1}{f_1} - \frac{1}{d_{o1}}$$

$$\frac{1}{d_{i1}} = \frac{1}{1.5\text{ cm}} - \frac{1}{1.7\text{ cm}} = \frac{1.7-1.5}{2.55\text{ cm}} = \frac{0.2}{2.55\text{ cm}}$$

$$\Rightarrow d_{i1} = \frac{2.55\text{ cm}}{0.2} = 12.8\text{ cm}$$

b) Because $d_{i1} > 0$, the image produced by the objective is real.

c) The image provided by the objective becomes the object for the eyepiece. Because the eyepiece is 14 cm from the objective and the image provided by the objective is 12.8 cm from the objective (therefore 1.2 cm from the eyepiece), the object distance for the eyepiece is 1.2 cm.

$d_{o2} = 1.2\text{ cm} \quad f_2 = 2.0\text{ cm}$

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \Rightarrow \frac{1}{d_{i2}} = \frac{1}{f_2} - \frac{1}{d_{o2}}$$

$$\frac{1}{d_{i2}} = \frac{1}{2.0\text{ cm}} - \frac{1}{1.2\text{ cm}} = \frac{1.2-2.0}{2.4\text{ cm}} = \frac{-0.8}{2.4\text{ cm}}$$

$$d_{i2} = \frac{-2.4\text{ cm}}{0.8} = -3.0\text{ cm}$$

d) Because $d_{i2} < 0$, the image produced by the eyepiece is virtual.

e) The image height produced by the objective is obtained by:

$$M_1 = \frac{h_{i1}}{h_{o1}} = -\frac{d_{i1}}{d_{o1}}$$

$$h_{i1} = -\frac{h_{o1}d_{i1}}{d_{o1}} = -\frac{1.0\text{ mm} \times 12.8\text{ cm}}{1.7\text{ cm}} = -7.5\text{ mm}$$

The object height for the eyepiece is equal to the image height provided by the objective:

$h_{o2} = -7.5\text{ mm}$

$$M_2 = \frac{h_{i2}}{h_{o2}} = -\frac{d_{i2}}{d_{o2}}$$

$$h_{i2} = -\frac{h_{o2}d_{i2}}{d_{o2}} = -\frac{-7.5\text{ mm} \times (-3.0\text{ cm})}{1.2\text{ cm}} = -19\text{ mm}$$

f) $$M_{tot} = \frac{h_{i2}}{h_{o1}} = \frac{-19\text{ mm}}{1.0\text{ mm}} = -19$$

g) Because $h_{i2} < 0$ (or $M_{tot} < 0$), the final image is inverted in relation to the object.

Section 5.4 The telescope

Textbook, p. 143

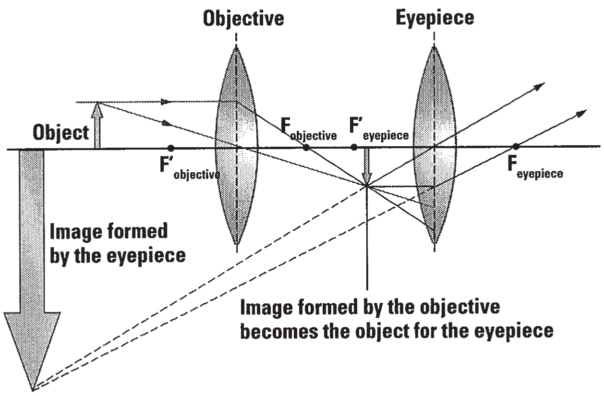
1. Refracting telescopes use lenses that refract light.

2. The fundamental difference between the refracting telescope and the reflecting telescope is the optical system used to capture the light. A refracting telescope uses a converging lens and a reflecting telescope uses a converging mirror.

3. The secondary mirror of a Newtonian telescope is positioned at 45° in relation to the primary mirror's principal axis in order to send the formed image toward the eyepiece in a direction that is perpendicular to the principal axis.

4. The world's biggest telescopes are reflecting telescopes because it is easier (fewer surfaces to polish) to make large mirrors than large lenses. Moreover, a large mirror can be supported from below while this is not the case for a lens through which light must pass.

5. Note: In the diagram in question 5, the object has a height of 10 mm and must be placed 31 cm from the centre of objective. To respect the distances represented in the textbook's diagram, the focal length of the objective is 15 mm and that of the eyepiece is 18 mm.



We first draw two principal rays passing through the objective in order to determine the the image position produced by the objective. This image becomes the object for the eyepiece because the rays reaching the eyepiece appear to be coming from this point. We can, therefore, draw principal rays originating from this second object. The rays emerging from the eyepiece diverge and the image provided by the eyepiece (the second image or final image) is virtual. We observe that it is inverted and larger than the initial object.

In the diagram, the initial object measures 1.0 cm: $h_{o1} = 1.0$ cm

The image provided by the objective (the first image) measures 0.9 cm and is inverted: $h_{i1} = -0.9$ cm

The image provided by the eyepiece (the second image) measures 5.5 cm and is inverted: $h_{i2} = -5.5$ cm

Magnification provided by the objective:

$$M_1 = \frac{h_{o1}}{h_{o1}} = \frac{-0.9\text{ cm}}{1.0\text{ cm}} = -0.9$$

Magnification provided by the eyepiece:

$$M_1 = \frac{h_{o2}}{h_{o2}} = \frac{-5.5\text{ cm}}{-0.9\text{ cm}} = 6.1$$

Total magnification:

$$M = \frac{h_{o2}}{h_{o2}} = \frac{-5.5\text{ cm}}{1.0\text{ cm}} = -5.5$$

We observe that $-0.9 \times 6.1 = -5.5$, therefore $M = M_1 \times M_2$.

Chapter 5
Applied Geometric Optics

Textbook, p. 147

1.

Camera	Human eye
Diaphragm	Iris
Objective	Cornea-crystalline lens optical system
Light-sensitive surface	Retina

2. You focus a camera by rotating the objective back and forth. In the case of the human eye, there is no mobile component. Focalization is done by accommodation of the crystalline lens, which changes form.

3. Virtual images: microscopes, reflecting and refracting telescopes.
Real images: camera, human eye.

4. *a)* Device 1 is a refracting telescope and Device 2 is a microscope.

   b) The first lens (on the left) is the objective while the second is the eyepiece. The objective produces a magnified real image. The eyepiece forms an even larger virtual image.

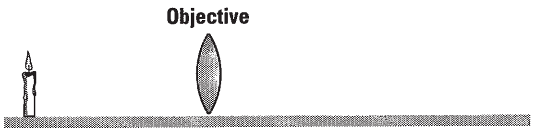
   c) The refracting telescope has a large focal length. As for the microscope, its objective is a lens with a very small focal length. The objective is placed very close to the object to be observed.

5. This is due to the fact that the primary mirror of reflecting telescopes does not have light passing through it, unlike the objective of refracting telescopes.

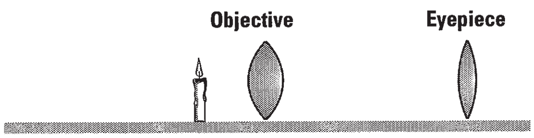
6.

Optical device	Components	Optical phenomenon (Reflection/Refraction)
Microscope	Objective	Refraction
	Eyepiece	Refraction
Reflecting telescope	Primary mirror	Reflection
	Secondary mirror	Reflection

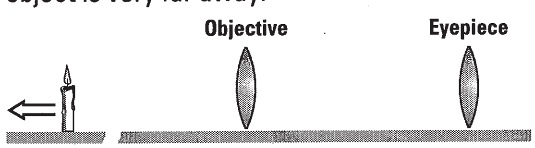
7. *a)* A camera has a converging lens.



b) A microscope has two converging lenses; the objective has a short focal length, therefore the lens is very rounded. The object is near the objective.



c) A refracting telescope has two converging lenses; the objective has a long focal length, therefore the lens is slightly rounded. The object is very far away.



UNIT 2

PRELIMINARY NOTIONS OF MECHANICS

Chapter 6 Frames of Reference

Textbook, p. 151 to 164

PRACTICE MAKES PERFECT

Section 6.1
The purpose of a frame of reference

Textbook, p. 154

1. A frame of reference is made up of a spatial reference, composed of an origin and three axes perpendicular to one another, to which a time reference can be added.

2. *a)* The observer on the moving sidewalk will see a vertical trajectory, perpendicular to the sidewalk's plane.

b) The observer on the ground, off the moving sidewalk, will see a parabolic trajectory.

3. *a)* *Note:* The axis of reference, determining the signs of velocity, points in the direction of the moving sidewalk.

v_{AB} = 1.3 m/s (velocity of the moving sidewalk)

v_B = -1.3 m/s (velocity of the person on the moving sidewalk)

The velocity of the person relative to the observer on the ground is:

$v_A = v_B + v_{AB} = 1.3\text{ m/s} - 1.3\text{ m/s} = 0\text{ m/s}$.

For the observer on the ground, off the moving sidewalk, the person on the sidewalk moves at a velocity of 0 m/s.

b) Everything happens as if the person were stationary relative to the ground. Therefore the observer on the ground, off the moving sidewalk, will see a vertical trajectory, perpendicular to the sidewalk's plane.

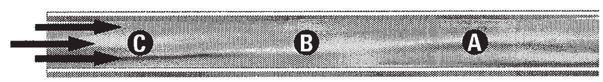
4. $t_{BC} = 18\text{ min} = 1080\text{ s}$

$d_{AB} = 800\text{ m}$

Velocity of the swimmer relative to the water $= v$

Velocity of the current relative to the shore $= v' = ?$

We assume that the log moves at the same velocity as the current.



Systematically, by using the notions covered in the section, we assert:

$t =$ Time taken by the log to go from point B to point A

$t_{BC} =$ Time taken by the person to swim from point B to point C

$t_{CB} =$ Time taken by the person to swim from point C to point B

$t_{BA} =$ Time taken by the person to swim from point B to point A

$d_{BC} =$ Distance between B and C = Distance between C and B.

The fact that the swimmer arrives at point A at the same time as the log is the central element of the problem. We can write that the time taken by the log to go from B to A is equal to the time taken by the swimmer to go from B to C to B to A:

$t = t_{BC} + t_{CB} + t_{BA}$.

The time taken by the log to go from point B to point A is:

$t = \dfrac{d_{AB}}{v'}$, where $d_{AB} = 800$ m. However, t is unknown.

From point B to point C, the person is swimming against the current. His speed in relation to the shore is equal to $v - v'$ (note that $v' < v$, if not, the person would not be able to swim up the river).

The time taken by the person to swim from point B to point C is:

$$t_{BC} = \frac{d_{BC}}{v - v'}$$

$\Rightarrow d_{BC} = (v - v') \times t_{BC}$, where $t_{BC} = 1080$ s.

From point C to point B, the person is swimming in the direction of the current. His velocity relative to the shore is equal to $v + v'$.

The time taken by the person to swim from point C to point B is:

$$t_{CB} = \frac{d_{BC}}{v + v'}.$$

By replacing the expression found previously for d_{BC}, we obtain:

$$t_{CB} = \frac{(v - v') \times t_{BC}}{v + v'}.$$

The time taken by the person to swim from point B to point A is:

$$t_{BA} = \frac{d_{AB}}{v + v'}.$$

Since $t = t_{BC} + t_{CB} + t_{BA}$, we obtain:

$$\frac{d_{AB}}{v'} = t_{BC} + \frac{v - v'}{v + v'} \times t_{BC} + \frac{d_{AB}}{v + v'}.$$

In this equation, d_{AB} and t_{BC} are known, and v and v' are unknown.

We transform the equation by finding a denominator common to the three terms on the right:

$$\frac{d_{AB}}{v'} = \frac{v + v'}{v + v'} \times t_{BC} + \frac{v - v'}{v + v'} \times t_{BC} + \frac{d_{AB}}{v + v'}$$

$$= \frac{v + v' + (v - v')}{v + v'} \times t_{BC} + \frac{d_{AB}}{v + v'}$$

$$\frac{d_{AB}}{v'} = \frac{2v}{v + v'} \times t_{BC} + \frac{d_{AB}}{v + v'}.$$

By isolating the two d_{AB} terms on the same side of the equation, we obtain:

$$\frac{d_{AB}}{v'} - \frac{d_{AB}}{v + v'} = \frac{2v}{v + v'} \times t_{BC}$$

$$d_{AB} \times \left(\frac{1}{v'} - \frac{1}{v + v'}\right) = \frac{2v}{v + v'} \times t_{BC}$$

$$d_{AB} \times \frac{(v + v') - v'}{v'(v + v')} = \frac{2v}{v + v'} \times t_{BC}$$

$$d_{AB} \times \frac{v}{v'(v + v')} = \frac{2v}{v + v'} \times t_{BC} \Rightarrow \frac{d_{AB}}{v'} = 2\, t_{BC}.$$

The velocity v no longer appears in the last equation. All that is left is the unknown velocity, which is the one we're looking for:

$$v' = \frac{d_{AB}}{2t_{BC}} = \frac{800 \text{ m}}{2 \times 1080 \text{ s}} = 0.370 \text{ m/s}.$$

Section 6.2
Inertial frames of reference

Textbook, p. 157

1. In the heliocentric frame of reference, the origin of the spatial reference is at the centre of the Sun. The three axes of reference (x, y and z) are perpendicular to one another and are directed toward distant stars that are considered fixed.

2. In the geocentric frame of reference, the Earth is driven by its rotation on its axis, from west to east, around an axis of rotation passing through its poles.

3. By definition, an inertial frame of reference is a frame of reference in which an isolated body is either at rest or in uniform rectilinear motion.

   Below are some examples of inertial frames of reference:

   – The terrestrial frame of reference for all experiments of short duration that are not influenced by the motion of the Earth's rotation.

   – The frame of reference associated with a car as long as it remains in uniform rectilinear motion.

4. Below are some examples of non-inertial frames of reference:

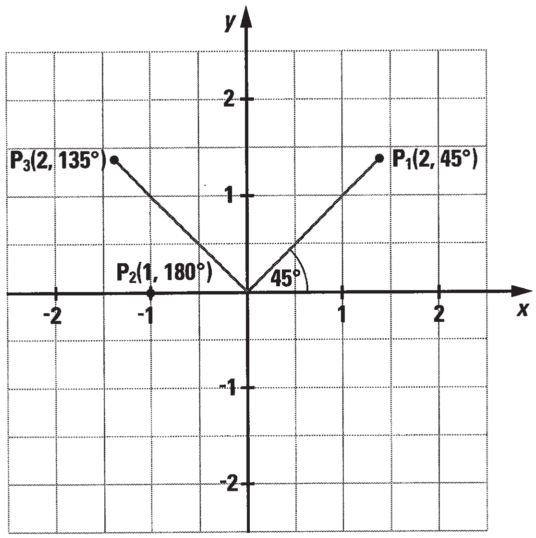
   – A frame of reference associated with a car making a turn.

- A frame of reference associated with an accelerating vehicle.
- A frame of reference associated with an amusement park ride in rotation.

Section 6.3
Coordinate systems

Textbook, p. 160

1.



2. Point P_1: $r_1 = 2, \theta_1 = 45° \Rightarrow x_1 = ?, y_1 = ?$

$x_1 = r_1 \times \cos\theta_1 = 2 \times \cos 45° = 1.4$

$y_1 = r_1 \times \sin\theta_1 = 2 \times \sin 45° = 1.4$

Point P_2: $r_2 = 1, \theta_2 = 180° \Rightarrow x_2 = ?, y_2 = ?$

$x_2 = r_2 \times \cos\theta_2 = 1 \times \cos 180° = -1$

$y_2 = r_2 \times \sin\theta_2 = 1 \times \sin 180° = 0$

Point P_3: $r_3 = 2, \theta_3 = 135° \Rightarrow x_3 = ?, y_3 = ?$

$x_3 = r_3 \times \cos\theta_3 = 2 \times \cos 135° = -1.4$

$y_3 = r_3 \times \sin\theta_3 = 2 \times \sin 135° = 1.4$

3. Point P_1 : $x_1 = 2, y_1 = 1 \Rightarrow r_1 = ?, \theta_1 = ?$

$r_1 = \sqrt{x_1^2 + y_1^2} = \sqrt{2^2 + 1^2} = 2.2$

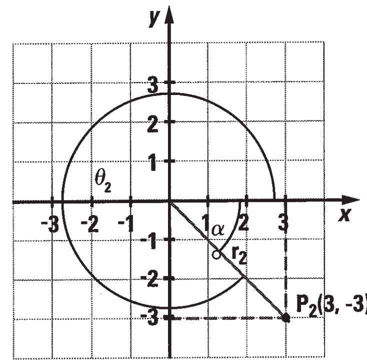
$\tan\theta_1 = \frac{y_1}{x_1} = \frac{1}{2} = 0.5 \Rightarrow \theta_1 = \tan^{-1}(0, 5) = 27°$

Therefore, $P_1(2.2, 27°)$

Point P_2: $x_2 = 3, y_2 = -3 \Rightarrow r_2 = ?, \theta_2 = ?$

$r_2 = \sqrt{x_2^2 + y_2^2} = \sqrt{3^2 + (-3)^2} = 4.2$

Point $P_2(3, -3)$ is in the fourth quadrant.



Let α, be the angle between the positive x-axis and the radius. Therefore,

$$\tan\alpha = \frac{|y_2|}{|x_2|} = \frac{3}{3} = 1 \Rightarrow \alpha = \tan^{-1}(1) = 45°$$

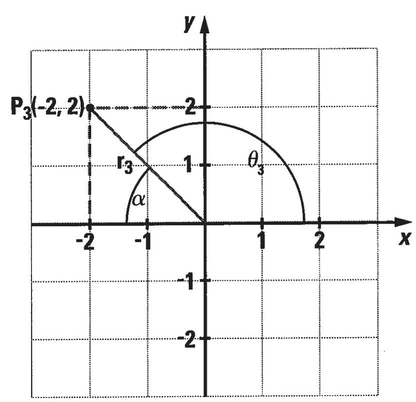
Therefore,

$\theta_2 = 360° - \alpha = 360° - 45° = 315°$, and $P_2(4.2, 315°)$

Point P_3: $x_3 = -2, y_3 = 2 \Rightarrow r_3 = ?, \theta_3 = ?$

$r_3 = \sqrt{x_3^2 + y_3^2} = \sqrt{(-2)^2 + (2)^2} = 2.8$

Point $P_3(-2, 2)$ is in the second quadrant.



Let α be the angle between the negative x-axis and the radius. Therefore,

$$\tan\alpha = \frac{|y_3|}{|x_3|} = \frac{2}{2} = 1 \Rightarrow \alpha = \tan^{-1}(1) = 45°$$

Therefore,

$\theta_3 = 180° - \alpha = 180° - 45° = 135°$, and $P_2(2.8, 135°)$.

4. *a)* The Cartesian coordinates can be read directly on the graph:

$P_1(5, 10)$ $P_2(4, -4)$ $P_3(-6, 9)$.

b) Point P_1

$r_1 = \sqrt{x_1^2 + y_1^2} = \sqrt{5^2 + 10^2} = 11$

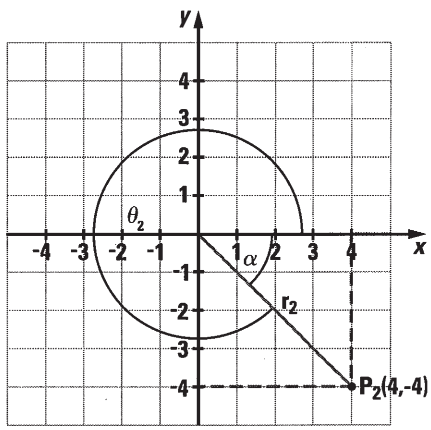
$\tan \theta_1 = \frac{y_1}{x_1} = \frac{10}{5} = 2 \Rightarrow \theta_1 = \tan^{-1}(2) = 63°$

Therefore, $P_1(11.2, 63°)$.

Point P_2

$r_2 = \sqrt{x_2^2 + y_2^2} = \sqrt{4^2 + (-4)^2} = 5.7$

Point P_2 is in the fourth quadrant.



Let α, be the angle between the positive x-axis and the radius. Therefore,

$\tan \alpha = \frac{|y_2|}{|x_2|} = \frac{4}{4} = 1 \Rightarrow \alpha = \tan^{-1}(1) = 45°$.

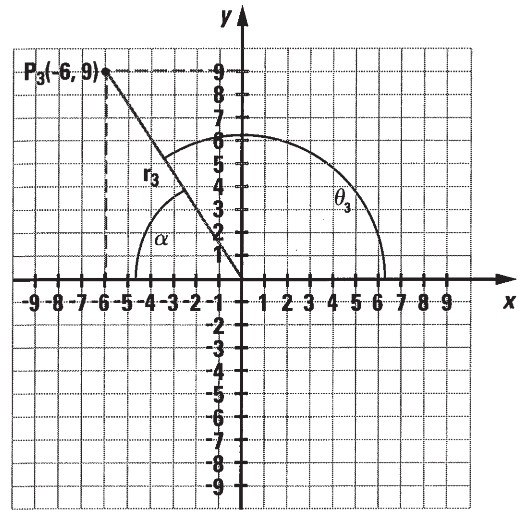
Therefore,

$\theta_2 = 360° - \alpha = 360° - 45° = 315°$ and $P_2(5.7, 315°)$.

Point P_3

$r_3 = \sqrt{x_3^2 + y_3^2} = \sqrt{(-6)^2 + 9^2} = 11$

Point P_3 is in the second quadrant.



Let α, be the angle between the negative x-axis and the radius. Therefore,

$\tan \alpha = \frac{|y_3|}{|x_3|} = \frac{9}{6} = 1.5 \Rightarrow \alpha = \tan^{-1}(1.5) = 56°$.

Therefore,

$\theta_3 = 180° - \alpha = 180° - 56° = 124°$ and $P_3(11, 124°)$.

Chapter 6
Frames of Reference

Textbook, p. 164

1. *a)* It is a terrestrial frame of reference.

b) This frame of reference is inertial because the duration of the experiment is very short compared to the Earth's diurnal rotation. This rotation, moreover, has practically no influence on the experiment

2. No. If you drop an object, for instance, it seems to fall vertically relative to the walls of the rail car, whether the rail car is stationary or moving at a constant velocity.

Chapter 7 Quantities and Units

Textbook, p. 165 to 182

PRACTICE MAKES PERFECT

Section 7.2
The International System of Units

Textbook, p. 168

1. Quantities: length, mass and speed.

   Units: metre (m), kilogram (kg), kilometre per hour (km/h).

2. *a)* A worker applies a force of 100 newtons to a crate.

   b) The table's mass is equal to 30 kg.

   c) The game lasted 45 s longer.

3. The label has an error because the gram (g) is a unit of mass, not weight. The label should instead read: "Net mass: 500 g."

4. • Length of the Nippur cubit = 0.5185 m

   • Number of Sumerian fingers in the Nippur cubit: 30

   • Number of Egyptian fingers in the Nippur cubit: 28

   Calculation of the length of the fingers:

   Length of a Sumerian finger

   $$= \frac{0.5185 \text{ m}}{30} = 0.017\,28 \text{ m}$$

   Length of an Egyptian finger

   $$= \frac{0.5185 \text{ m}}{28} = 0.018\,52 \text{ m}.$$

   Calculation of the number of fingers in one metre:

   Number of Sumerian fingers in one metre

   $$= \frac{1 \text{ m}}{0.017\,28 \text{ m}} \approx 58$$

   Number of Egyptian fingers in one metre

   $$= \frac{1 \text{ m}}{0.018\,52 \text{ m}} \approx 54.$$

Section 7.3
Fundamental standards of the basic units of mechanics

Textbook, p. 171

1. No. The material standard of the metre has not been in use since 1960.

2. The only material standard of an SI unit base unit still used today is the kilogram.

3. To define the metre, it has been agreed that the speed of light in a vacuum should be set at $c = 299\,792\,458$ m/s.

4. This definition has been abandoned because of the Earth's rotational irregularities.

Section 7.4
Derived units in the International System

Textbook, p. 172

1. 1 kWh = 1000 W × 1 h = 1000 W × 3600 s

   = 3 600 000 W • s

   In terms of SI base units: 1 W = 1 kg m^2 s^{-3}.

   Therefore, 1 kWh = 3 600 000 W • s = 3 600 000 kg m^2 s^{-3} × 1 s = 3 600 000 kg m^2 s^{-2}.

   Because this combination of units corresponds to the joule (*see Table 5 on page 172 in the textbook*), we can therefore state that 1 kWh = 3 600 000 J.

2. $W = F \times \Delta s$

   Unit of (W) = unit of (F) × unit of (Δs) = N × m = (kg m s^{-2}) × m = kg m^2 s^{-2}

3. Weight = mass × gravitational acceleration or $F_g = mg$

   (*See the* Review *on page 13 in the textbook.*)

Unit of weight = (unit of mass) × (unit of gravitational acceleration)

$= kg \times m\,s^{-2} = kg\,m\,s^{-2}$

Because the unit of force is the kilogram-metre per second squared, or kg m s^{-2} (*see Table 5 on page 172 in the textbook*), we can conclude that weight is a force.

4. $p = \frac{F}{A} \Rightarrow \text{Unit of } (p) = \frac{\text{Unit of } (F)}{\text{Unit of } (A)} = \frac{N}{m^2}$

Unit of $(p) = kg\,m^{-1}\,s^{-2}$

Section 7.5 Multiples and submultiples of units

Textbook, p. 175

1. $t = 250$ s

a) $t = 250\text{ s} \times \frac{10^6\ \mu s}{1\text{ s}} = 2.50 \times 10^8\ \mu s$

$t = 250\ 000\ 000\ \mu s$

b) $t = 250\text{ s} \times \frac{10^3\text{ ms}}{1\text{ s}} = 2.50 \times 10^5\text{ ms}$

$t = 250\ 000$ ms

c) $t = 250\text{ s} \times \frac{10^{-3}\text{ ks}}{1\text{ s}} = 2.50 \times 10^{-1}\text{ ks}$

$t = 0.250$ ks

d) $t = 250\text{ s} \times \frac{10^{-6}\text{ Ms}}{1\text{ s}} = 2.50 \times 10^{-4}\text{ Ms}$

$t = 0.000\ 250$ Ms

2. $E = 150\text{ MJ} = 150 \times 10^6\text{ J} = 1.50 \times 10^8\text{ J}$

$1\text{ mJ} = 10^{-3}\text{ J}$

Therefore,

$E = 1.50 \times 10^8\text{ J} \times \frac{1\text{ mJ}}{10^{-3}\text{ J}} = 1.50 \times 10^{11}\text{ mJ}$.

3. $m = 10.2\ \mu g = 10.2 \times 10^{-6}\text{ g} = 1.02 \times 10^{-5}\text{ g}$

$1\text{ kg} = 1000\text{ g}$

$m = 1.02 \times 10^{-5}\text{ g} \times \frac{1\text{ kg}}{10^3\text{ g}} = 1.02 \times 10^{-8}\text{ kg}$

4. $A = 23.5\text{ cm}^2$ $1\text{ m} = 100\text{ cm}$

$A = 23.5\text{ cm}^2 \times \left(\frac{1\text{ m}}{100\text{ cm}}\right)^2 = 2.35 \times 10^{-3}\text{ m}^2$

5. *a)* $v = 25\frac{km}{h} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = 6.9\frac{m}{s}$

b) $v = 150\frac{km}{h} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = 41.7\frac{m}{s}$

c) $v = 2.0\frac{m}{s} \times \frac{1\text{ km}}{1000\text{ m}} \times \frac{3600\text{ s}}{1\text{ h}} = 7.2\frac{km}{h}$

d) $v = 50\frac{m}{s} \times \frac{1\text{ km}}{1000\text{ m}} \times \frac{3600\text{ s}}{1\text{ h}} = 180\frac{km}{h}$

6. Unit of $(E_k) = J = kg\,m^2\,s^{-2}$

Unit of $\frac{1}{2}mv^2$ = unit of (m) × unit of (v^2)

$= kg \times \left(\frac{m}{s}\right)^2 = kg\,m^2\,s^{-2}$

The formula for kinetic energy is therefore homogeneous.

7. *a)* $l = 1$ micrometre

b) $t = 1$ femtosecond

c) $m = 1$ microgram

8. *a)* $l = 1$ km

b) $E = 1$ MJ

c) $P = 1$ GW

9. Unit of (T): s

Unit of (g): m/s^2

$\text{Unit of } \sqrt{\frac{l}{g}} = \sqrt{\frac{\text{Unit of } (l)}{\text{Unit of } (g)}} = \sqrt{\frac{m}{m/s^2}} = \sqrt{m \times \frac{s^2}{m}}$

$= \sqrt{s^2} = s$

The formula is homogeneous.

10. $k = \frac{F_r}{\Delta x}$

$\text{Unit of } k = \frac{\text{Unit of } (F_r)}{\text{Unit of } (\Delta x)} = \frac{N}{m}$

The unit of the spring constant *k* is the newton per metre (N/m).

Section 7.6
Scalar quantities and vector quantities

Textbook, p. 177

1. Scalar quantities are completely defined by a real number and a unit of measure. Vector quantities are characterized by direction and also by magnitude and unit of measure.

2. These are said to be vector quantities because they require a vector to be completely defined.

3. Scalar quantities: length, area, volume, mass, time, work, energy, power, frequency, period, angle, index of refraction, convergence, etc.
Vector quantities: position, displacement, speed, acceleration, force, etc.

4. A vector is characterized by:
   - its magnitude, which represents its quantity
   - its direction, which is that of the line and arrowhead carrying the vector

Chapter 7
Sizes and Units

Textbook, p. 182

● 1.

Factor	Name	Symbol
10^{21}	zetta	Z
10^{1}	deca	da
10^{-15}	femto	f
10^{-3}	milli	m
10^{9}	giga	G
10^{-1}	deci	d

● 2.

Information	Basic SI unit
Unit based on a property of the caesium 133 atom	second
Unit defined in relation to a distance travelled by light	metre
Only unit defined in relation to a material standard	kilogram
Unit that has been modified four times since 1889	metre
Most precisely known unit in the SI	second

● 3.

Quantity	Scalar/vector
Force	Vector
Mass	Scalar
Velocity	Vector
Acceleration	Vector
Energy	Scalar

■ 4. Given that mass is a scalar quantity, and velocity is a vector quantity, their product is a vector quantity. When a scalar and a vector are multiplied, the result is a vector.

■ 5. $p = m \times v \Rightarrow$ Unit of (p) = Unit of $(m) \times$ Unit of (v)

$$\text{Unit of } (p) = \text{kg} \times \frac{\text{m}}{\text{s}} = \text{kg m s}^{-1}$$

The unit for momentum is the kilogram-metre per second (kg m s^{-1}).

■ 6. $F = G\dfrac{m_1 m_2}{r^2} \Rightarrow G = \dfrac{Fr^2}{m_1 m_2}$

$$\text{Unit of } G = \frac{\text{Unit of } (F) \times \text{Unit of } (r^2)}{\text{Unit of } (m_1) \times \text{Unit of } (m_2)} =$$

$$= \frac{\text{N} \times \text{m}^2}{\text{kg} \times \text{kg}} = \frac{(\text{kg m s}^{-2}) \times \text{m}^2}{\text{kg} \times \text{kg}} = \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$$

The unit of the gravitational constant is: $m^3\ kg^{-1}\ s^{-2}$.

◆ 7. $m = 1$ kg

Diameter of the cylinder (D) = height of the cylinder (h) $= 39 \times 10^{-3}$ m

$$\text{Relative density } (\rho) = \frac{\text{Mass } (m)}{\text{Volume } (V)}$$

$$V = \pi \times r^2 \times h = \pi \times \left(\frac{D}{2}\right)^2 \times h$$

$$= \pi \times \left(\frac{39 \times 10^{-3}\,\text{m}}{2}\right)^2 \times 39 \times 10^{-3} m$$

$$= 4.66 \times 10^{-5}\ \text{m}^3$$

$$\rho = \frac{m}{V} = \frac{1\,\text{kg}}{4.66 \times 10^{-5}\ \text{m}^3} = 2.2 \times 10^4\ \frac{\text{kg}}{\text{m}^3}$$

$$\rho = 2.15 \times 10^4\ \frac{\text{kg}}{\text{m}^3} \times \frac{1000\ \text{g}}{1\ \text{kg}} \times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 = 22\ \frac{\text{g}}{\text{cm}^3}$$

★ 8. If we look closely at the base units of energy and power, we observe they are the same, except for the exponent of the second(s). We can therefore