Chapter 1: Vectors

Question:
How can we model (mathematically) the motion of objects through space and time?

Definitions

Quantities used in Physics can be categorized as either scalars or vectors

A scalar is defined by a magnitude (size) only

A vector is defined by a magnitude and a direction

Vectors and scalars are mutually exclusive, i.e. no quantity can be both.

Examples:

5.0 km: Scalar

5.0 km, (North): Vector

Some common quantities used in Physics (or science in general):

Scalars	Vectors
distance speed time Volume Energy	displacement velocity acceleration Force

Representing Vectors

~~*Graphically – drawing vectors*~~

~~Arrows~~ are used to represent vector quantities

Length of arrow: magnitude } to scale

Where the arrow points: direction

Example:

1) Draw the vector: 25 km to the left 1 cm = 5 km

2) Consider the vector below. (assume 1.0 cm = 20 N). What does this vector represent?

$2.5\text{cm} \times \frac{20\text{N}}{1.0\text{cm}} = 50\text{N}$

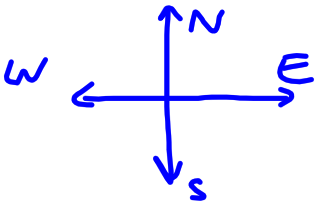
50N, down

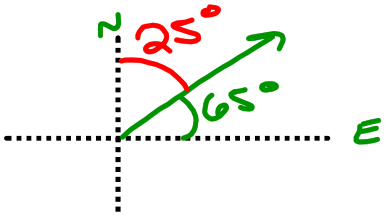
Using Magnitude and Direction

We can represent a vector by giving its magnitude and its direction. The magnitude must have units.

The direction can be expressed in 2 ways:

- using the cardinal points:

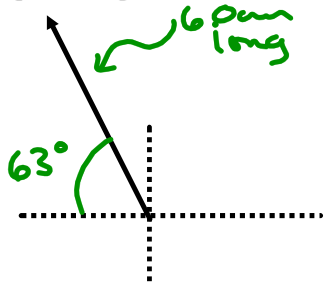




Notation [E 65° N] means: start from E, go 65° towards N

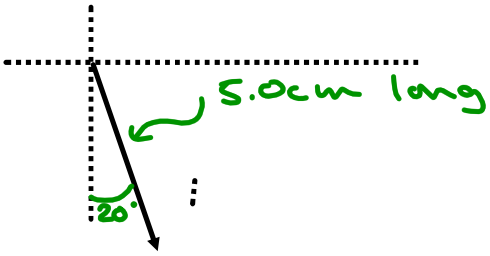
[N̄25°E]

1) Draw the vector:
6.0 cm [W 63 °N]

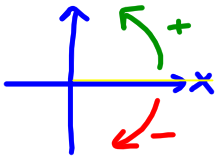


2) Using a scale of 5.0 km = 1.0 cm,
draw the vector: 25 km [S 20° E]

$25\,km \times \frac{1.0\,cm}{5.0\,km} = 5.0\,cm$

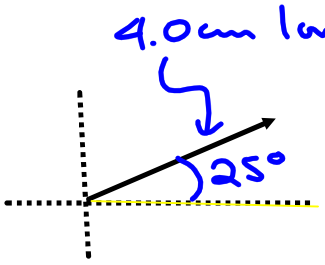


- using an angle on the Cartesian plane.



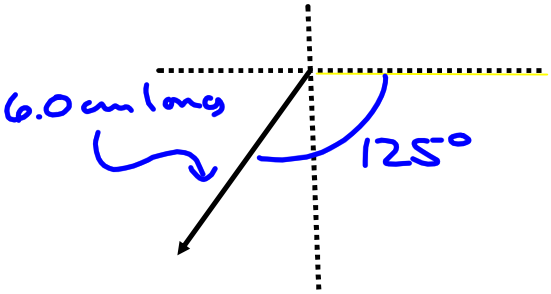
Always measure the angle from the "positive x-axis" (like East!)

1) Draw the vector: 4.0 m/s at 25°
(Scale: 1.0 cm = 1.0 m/s)



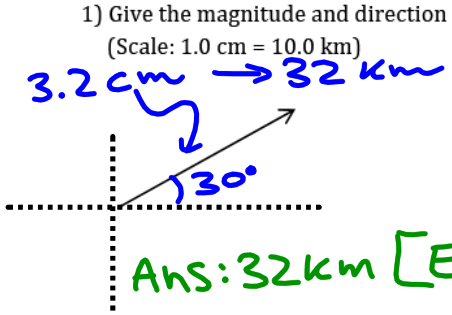
2) Draw the vector: 60.0 N, -125°

10 N = 1 cm

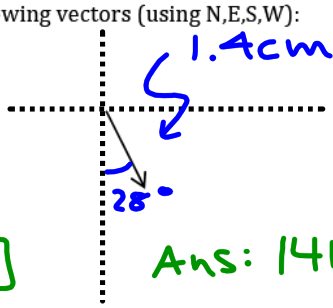


Going back and forth between the different representations

1) Give the magnitude and direction of the following vectors (using N,E,S,W):
(Scale: 1.0 cm = 10.0 km)

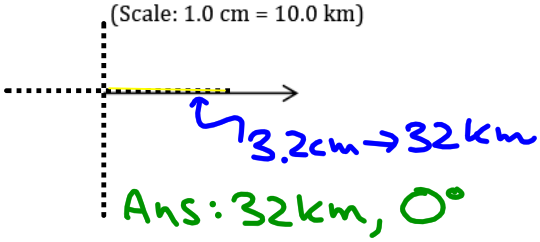


Ans: 32km [E30°N]

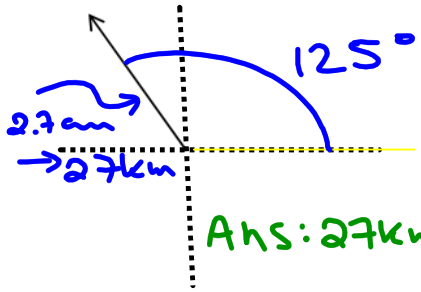


Ans: 14km [S 28° E]

2) Give the magnitude and direction of the following vectors (using angle on Cartesian plane):
(Scale: 1.0 cm = 10.0 km)



Ans: 32km, 0°



Ans: 27km, 125°

Vector Addition

Vector notation: To show that a quantity is a vector, we write it with an "arrow on top" $\vec{A}$, $\vec{F}$

When we add 2 or more vectors, the result is another vector.

The vector obtained after adding various vectors is called the resultant ($\vec{R}$).
We can think of the resultant as the vector that could replace the combination of the other vectors.

Adding vectors graphically – your drawing must be to scale

Using the Head to Tail Method

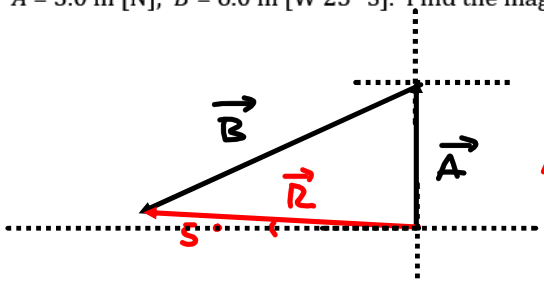
- Draw vectors one after the other.
  (Make sure the magnitudes and directions (angles) are to scale!)
- Draw the resultant vector
  (The resultant is drawn from the starting point to the ending point)
- Measure the magnitude and direction of the resultant.

Note: The order of addition does not matter.

Examples:

1. Draw the vector resulting from the addition of the two vectors below.

2. Consider $\vec{A}$ = 3.0 m [N], $\vec{B}$ = 6.0 m [W 25° S]. Find the magnitude and direction of $\vec{A}+\vec{B}$.



(1.0 m = 1 cm)

Ans: 5.5 m [W 4° N]

Simple operations on vectors

Consider the vector $\vec{A}$ = 4.0 N [E 40°N]

Multiplying a vector by a constant: multiply the magnitude by the constant, direction is the same

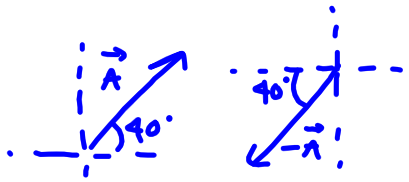
Ex: $2\vec{A}$ = 8.0 N [E 40° N]

Taking the negative of a vector: Same magnitude, but in the opposite direction

Ex: $-\vec{A}$ = 4.0 N [W 40° S]

$\vec{A}$: E 40° N

$-\vec{A}$: W 40° S



A few more examples

Consider the vectors below: Scale: 1.0 cm = 10.0 km

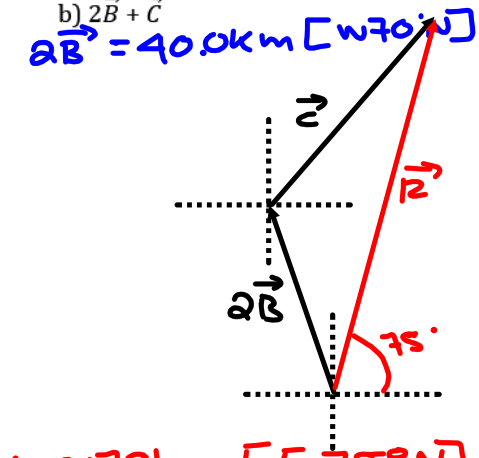
$\vec{B}$ = 20.0 km [W 70° N]

$\vec{C}$ = 50.0 km [N 40° E]

Find the resultant vector of

b) $2\vec{B} + \vec{C}$

$2\vec{B}$ = 40.0km [W70°N]

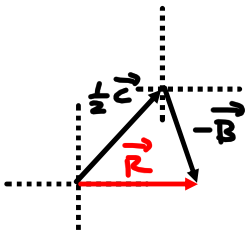


Ans: 78km [E 75°N]

c) $\frac{1}{2}\vec{C} - \vec{B}$

$\frac{1}{2}\vec{C}$ = 25.0km [N40°E]

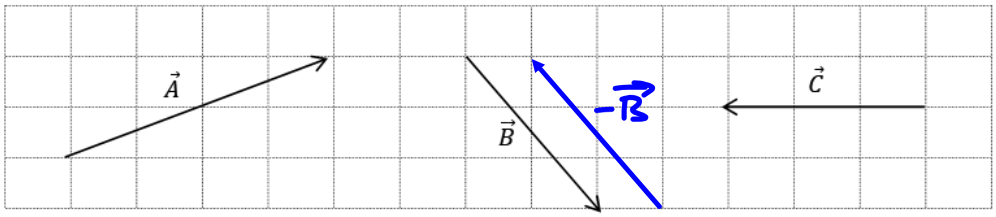
$-\vec{B}$ = 20.0km [E70°S]



Ans: 26km [E]

Adding Vectors on a grid...

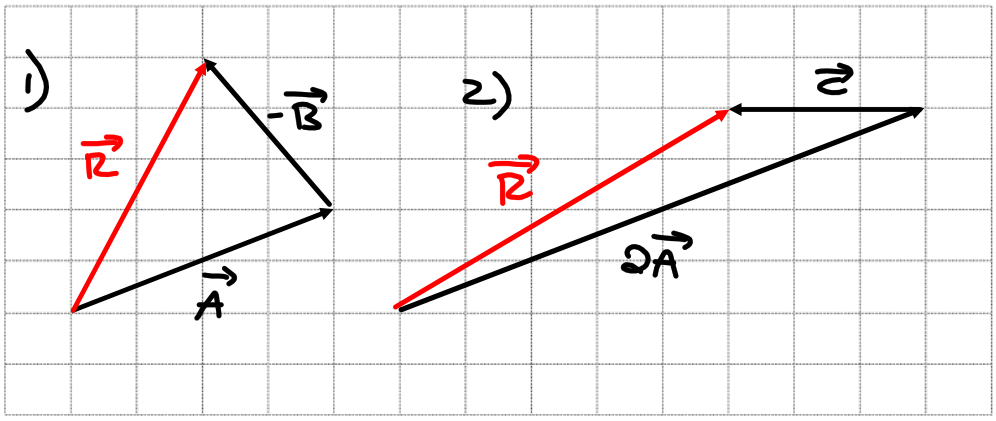
Consider the vectors below:



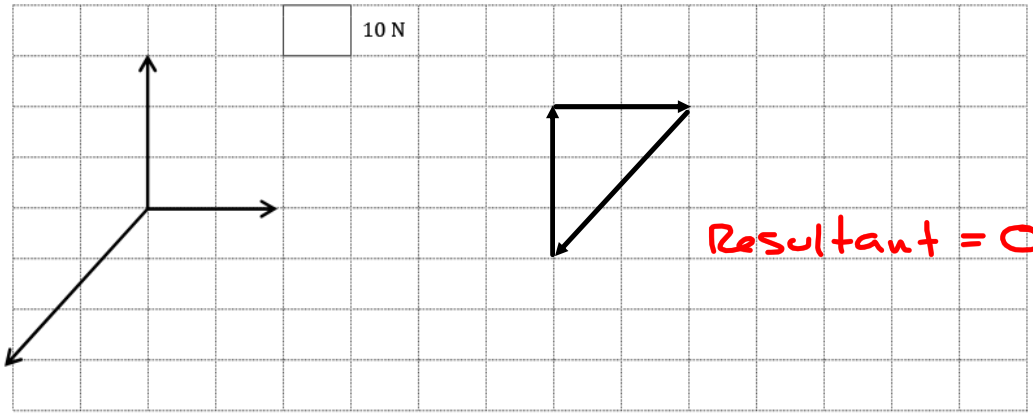
Draw the resultant vector from the following additions:

1) $\vec{A} - \vec{B}$

2) $2\vec{A} + \vec{C}$



What is the sum of the vectors show below?



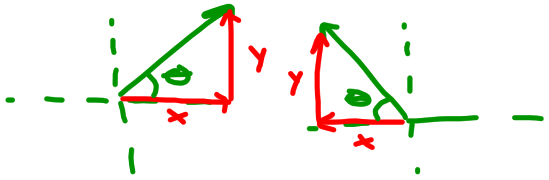
~~Method 2 – Adding Vector by Components~~

Expressing Vectors Using Components

Reminder: We can represent vectors using

- a magnitude and an ~~angle with N,E,S,W~~
- a magnitude and ~~angle from the positive x-axis~~

We can also represent vectors using the ~~x and y components~~ (or coordinate)
Note: We use the tail of the vector as the origin.



BE CAREFUL ~~WITH SIGNS~~!!!

x : ⊕ = right
⊖ = left

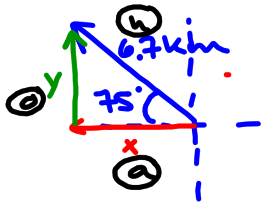
y : ⊕ = up
⊖ = down

*add the signs yourself

SOH CAH TOA

Examples:

1) What are the x and y components for the vector 6.7 km [W75°N]



(x) $\cos 75° = \frac{x}{6.7\,km}$

$x = (6.7\,km)(\cos 75°)$

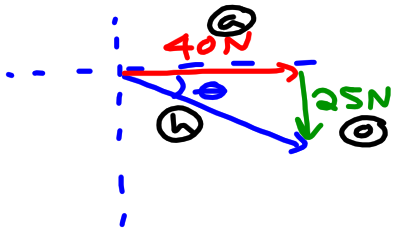
$x = 1.73\,km$

(y) $\sin 75° = \frac{y}{6.7\,km}$

$y = (6.7\,km)(\sin 75°)$

$y = 6.47\,km$

2) A force vector has the following x and y coordinates: (40 N, -25N). Express this vector using a "magnitude and a direction.



$mag = \sqrt{(25\,N)^2 + (40\,N)^2}$ (pythag.)

$= 47.2\,N$

Angle: $\tan\theta = \frac{25\,N}{40\,N}$

$\theta = \tan^{-1}\left(\frac{25\,N}{40\,N}\right)$

$= 32°$

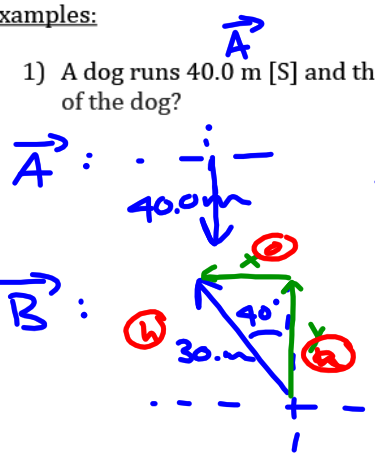
Ans: 47.2N [E32°S]

Adding Vectors by Components

1. Split the vectors into x and y components
2. Add the x components together (this gives the x component of the resultant)
   Add the y components together (this gives the y component of the resultant)
3. Sketch the x and y for the resultant
4. Calculate the magnitude and direction.

Examples:

1) A dog runs 40.0 m [S] and then 30.0 m [N40°W]. What is the resultant displacement of the dog?



$x: 0$

$y: -40m$

$x: \sin 40° = \frac{x}{30m}$

$x = (30m)(\sin 40°)$

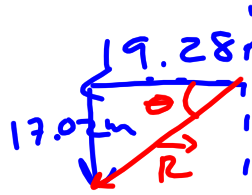
$x = -19.28m$

$y: \cos 40° = \frac{y}{30m}$

$y = (30m)(\cos 40°)$

$= +22.98m$

	x	y
$\vec{A}$	0	−40 m
+ $\vec{B}$	−19.28	+22.98 m
$\vec{R}$	−19.28	−17.02 m



$Mag = \sqrt{(19.28m)^2 + (17.02m)^2}$

$= 25.7m$

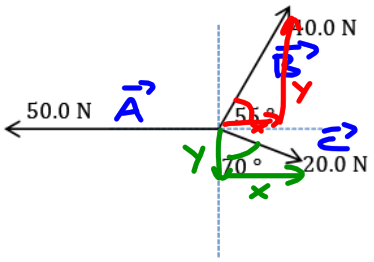
$\tan\theta = \frac{17.02m}{19.28m}$

$\theta = \tan^{-1}\left(\frac{17.02m}{19.28m}\right)$

$= 41.4°$

Ans: 25.7m [W41.4°S]

2) Three forces are simultaneously applied to an object, as is illustrated below. What is the resultant force on the object?



$\vec{A}$: $x = -50.0\,N$
$y = 0$

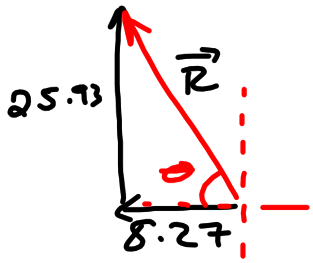
$\vec{B}$: $x \rightarrow \cos 55^\circ = \frac{x}{40.0\,N}$
$x = 40.0\,N \cos 55^\circ$
$= +22.94\,N$

$y \rightarrow \sin 55^\circ = \frac{y}{40.0\,N}$
$y = 40.0\,N \sin 55^\circ$
$= +32.77\,N$

$\vec{C}$: $x \rightarrow \sin 70^\circ = \frac{x}{20.0\,N}$
$x = 20.0\,N \sin 70^\circ$
$x = +18.79\,N$

$y \rightarrow \cos 70^\circ = \frac{y}{20.0\,N}$
$y = 20.0\,N \cos 70^\circ$
$= -6.84\,N$

$\vec{A}\,(-50\ ,\ 0\ )\,N$
$\vec{B}\,(22.94,\ 32.77)\,N$
$\vec{C}\,(18.79,\ -6.84)\,N$
$\vec{R}\,(-8.27,\ 25.93)\,N$



$mag = \sqrt{(25.93N)^2 + (8.27N)^2}$
$= 27.2\,N$

$\tan\theta = \frac{25.93N}{8.27N}$

$\theta = \tan^{-1}\left(\frac{25.93}{8.27}\right)$
$= 72.3^\circ$

Ans: 27.2N [W72.3°N]

Some More Word Problems (A few classics)

Distance and Displacement

Distance is the length of the path traveled (add all magnitudes)

Distance is a scalar, it does not have a direction.

Displacement is the change in position (from starting point to ending point)

Note: Position is the location of an object

Displacement is a vector, it ~~must be expressed~~ using a magnitude and a direction.

Examples

1) Consider a cat chasing a mouse. The cat runs 4.0 m to the right, 2.0 m to the left, 1.0 m to the right and 5.0 to the left.

   a) What is the distance covered by the cat?

   distance = 4.0m + 2.0m + 1.0m + 5.0m
   = 12.0m

   b) What is the displacement of the cat?

   displacement: +4.0m + (-2.0m) + (1.0m) + (-5.0m)

   ← (−) → (+)

   = -2.0m
   ↑ to the left

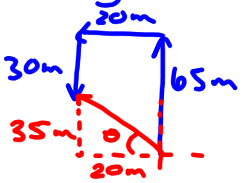
2) During a practice, football players run 65 m [N], shuffle 20 m [W] and run 30 m [S].

   a) What distance do the players cover?

   distance = 65m + 20m + 30m
   = 115m

   b) What is the displacement of the players?

① method Sketch (no angles!)



mag = $\sqrt{(20m)^2 + (35m)^2}$
= 40.3m

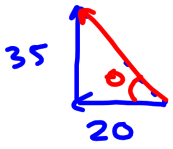
$\tan\theta = \frac{35m}{20m}$

$\theta = \tan^{-1}\left(\frac{35m}{20}\right)$
= 60.3°

Ans: 40.3m [W60.3°N]

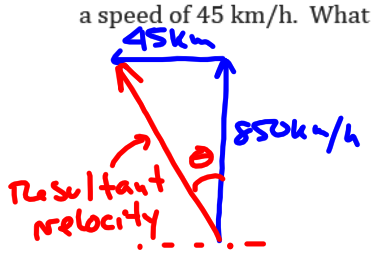
② method By component

(0, 65)
(-20, 0)
+ (0, -30)
(-20, 35)



Wind/Current Problems

1) A plane is flying due [N] at a speed of 850 km/h. The wind is blowing from the [E] at a speed of 45 km/h. What is the resultant velocity of the airplane?



$mag = \sqrt{(45km/h)^2 + (850km/h)^2}$

$= 851\ km/h$

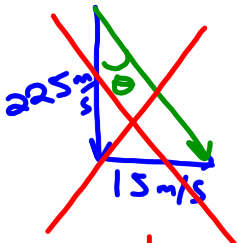
$\theta = \tan^{-1}\left(\frac{45km/h}{850km/h}\right)$

$= 3°$

Ans: 851km/h [N3°W]

2) A plane needs to reach a destination located directly south of its departure. The plane's engine can provide a maximum speed of 225 m/s. The wind is blowing at 15 m/s in the [E] direction. What heading should the plane take in order to reach its destination?

direction?



No! you won't end-up [S]

Compensate for the wind

← Resultant Velocity = South!

$\sin\theta = \frac{15\ m/s}{225\ m/s}$

$\theta = \sin^{-1}\left(\frac{15\ m/s}{225\ m/s}\right)$

$= 3.8°$

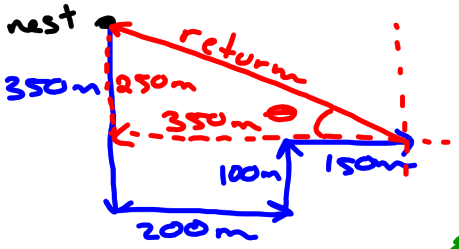
Ans: [S3.8°W]

"Return" Vector (back to starting point)

The "return" vector has the same magnitude as the resultant vector, but it is in the opposite direction.

Ex: A bird leaves its nest and flies horizontally. The bird flies 350 m [S], 200 m [E], 100 m [N] and then 150 m [E]. From this point, the bird flies back to its nest. What is the displacement of the bird as it returns to its nest?

Method ① - sketch (no angle)



Method ② - components

(0, -350) m
(200, 0) m
(0, 100) m
(150, 0) m

$\vec{R}$ (350, -250) m

opposite direction

Return (-350, 250) m

$$mag = \sqrt{(350m)^2 + (250m)^2}$$
$$= 430m$$

$$\theta = \tan^{-1}\left(\frac{250m}{350m}\right)$$
$$= 35.5°$$

Ans: 430m [W 35.5° N]