Chapter 3: Significant Figures and Uncertainty

Significant Figures

Significant figures (or digits) are important when you are working with data that has been measured. It is important to consider them when you are doing some calculations using numbers obtained from measurements.

What is the point of sig figs? Knowing about sig figs allows you to ROUND your answer properly.

What are significant figures?

For any given measurement, some digits (or figures) are significant, while some are non-significant. Significant figures always indicate precision.

Here are the rules for determining whether or not digits are significant:

1. Non-zero digits (from 1-9) are always significant.

   Examples:

   a) 68 cm has 2 significant figures.

   b) 2597 m has 4 significant figures.

2. Zeros are significant when they are

   a. "sandwiched" in between two other significant digits

      Examples:

      a) 72 502 m has 5 significant figures.

      b) 206 s has 3 significant figures.

      c) 4005 cm has 4 significant figures.

   b. to the right of both the decimal place and another significant digit

      Examples:

      a) 75.00 minutes has 4 significant figures.

      b) 20.0 cm has 3 significant figures.

      c) 20.010 cm has 5 significant figures.

3. Zeros are not significant when used solely for spacing the decimal point (placeholders)

i.e.: leading zeros (before decimal, small number) are not significant

$0.00025 \text{ kg} = 2.5 \times 10^{-4} \text{ kg}$

trailing zeros (at the end of large number) are not significant

$28\,000 \text{ L} = 2.8 \times 10^{4} \text{ L}$

Examples:

a) 0.0067 m has 2 significant figures.

b) 25 000 m has 2 significant figures.

c) 2.36×10^4 s has 3 significant figures.

d) 0.0250 m has 3 significant figures.

e) 230.00 s has 5 significant figures.

4. Numbers that are not measurements (for example, constants in a formula or values that have been counted) have an infinite number of significant figures. We basically ignore them when we are trying to decide how many significant figure we need to round to.

Example:

a) There are 41 bunnies in a cage.

b) Near the surface of the Earth, the acceleration due to gravity is $\boxed{9.8 \text{ m/s}^2.}$ not 2SF

c) The formula for finding the area of a triangle: $A = \dfrac{b \times h}{2}$ not 1SF

Rounding using Significant Figures (sig figs)

Instead of rounding to a specific number of decimal places, will will be rounding to a specific number of sig figs.

Examples:

1. Round 23.5824 m to
   a. 2 sig figs: 24 m
   b. 3 sig figs: 23.6m
   c. 4 sig figs: 23.58m

2. Round 0.005326 m to
   a. 1 sig figs: 0.005 m
   b. 2 sig figs: 0.0053 m
   c. 2 sig figs, (in scientific notation): 5.3×10^{-3} m
   d. 3 sig figs: 0.00533 m

3. Round 324 267 m to
   a. 2 sig figs: 320 000 m
   b. 2 sig figs (in scientific notation): 3.2×10^{5} m
   c. 4 sig figs: 324 300 m
   d. 4 sig figs, (in scientific notation): 3.243×10^{5} m

4. Write the quantity 280 m so that it has   280   280.0
   a. 1 sig figs: 300 m
   b. 2 sig figs: 280 m
   c. 3 sig figs: 280. m or 2.80×10^{2} m
   d. 4 sig figs: ______________

The rule for addition and subtraction with significant figures

When **adding** or **subtracting** round the answer to the **least number of decimal places**.

Examples:

a) 32.3 km + 51 km = 83.3 km → 83 km (0 dec)
   1 dec   0 dec

b) 6.235 s - 2.54 s + 2.05 s = 5.745 s → 5.75 s (2 dec)
   3 dec   2 dec   2 dec

c) 26.87 cm + 2.004 cm + 61.025 cm = 89.899 cm → 89.90 cm (2 dec)
   2 dec   3 dec   3 dec

→ everything else!

The rule for ~~multiplication and division~~ with significant figures

When **multiplying** or **dividing** with numbers round the answer to the ~~least number of sig figs~~ **i.e.** round off your answer to **match the same number of significant figures as your measurement with the least number of significant figures.**

*only round once!

Examples:

a) $0.0880\ m/s^2 \times 3.4\ s$ = $0.2992\ \frac{m}{s}$ → $0.30\ m/s$ (2 SF)
3 SF, 2 SF

b) $101\ s \times 6.0\ m/s$ = $606\ m$ → $610\ m$ or $6.1 \times 10^2\ m$ (2 SF)
3 SF, 2 SF

c) $45.0\ m \div 3.50\ m/s$ = $12.85714...\ s$ → $12.9\ s$ (3 SF)
3 SF, 3 SF

round only once!

d) $6.35\ cm \times 3.04\ cm \times 25\ cm$ = $482.6\ cm^3$ → $480\ cm^3$ or $4.8 \times 10^2\ cm^3$ (2 SF)
3 SF, 3 SF, 2 SF

e) $\frac{2.588\ cm \times 6.80\ cm}{18.09\ cm}$ = $0.972825...\ cm$ → $0.973\ cm$ (3 SF)
4 SF, 3 SF, 4 SF

f) The base of a triangle measures 2.39 cm and its height measures 8.53 cm. What is the area of this triangle?

$A = \frac{b \times h}{2} = \frac{(2.39\ cm)(8.53\ cm)}{2}$ (3 SF, 3 SF)

$= 10.2\ cm^2$ (3 SF)

g) What is the area of a circle that has a radius of 3.9 cm?

$A = \pi r^2$

$= \pi (3.9\ cm)^2$ (2 SF)

$= 48\ cm^2$ (2 SF)

h) What is the weight of a 7.89 kg mass?

$F_g = mg$

$= (7.89\ kg)(9.8\ \frac{N}{kg})$ (3 SF)

→ constant = disregard SF

$= 77.3\ N$ (3 SF)

i) A car accelerates at a rate of $0.025\ m/s^2$ for 4.50 s. What is the change in velocity of the car during this time?

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

$\Delta \vec{v} = \vec{a} \Delta t$

$= (0.025\ \frac{m}{s^{\cancel{2}}})(4.50\ \cancel{s})$ (2 SF, 3 SF)

$= 0.11\ \frac{m}{s}$ (2 SF)

Multi-Step Problems

When a problem involves both addition and multiplication, each rule has to be applied separately.

How do you know if a problem is multistep?

- If you have to apply the 2 different rules (rouding to decimals for + and -, and rounding to S.F. for × and ÷), you are dealing with a multistep problem.
- Doing a series of multiplicatiion is NOT a multistep porblems (as far as sig figs are concerned). ex: $\sqrt{\dfrac{2\times 6.3\times 5}{7.81}}$ ← not a multistep

So how do we deal with multistep problems?

- Keep one EXTRA sig fig or EXTRA decimal for your intermediate (not final) answer. This avoids "overrounding".
- Keep track of the "extra" using the dot on top notation.

Ex: 25.668 cm to 3 sig figs is: 25.7 cm

25.668 with 3 sig figs and an "extra" is : $25.6\dot{7}$ cm
↳ still 3 SF (extra doesn't count)

Examples:

1) *distance* = (4.56 s + 12.678 s)(3.99 m/s)

2 dec 3 dec

$= (17.23\dot{8}\ s)(3.99\ \frac{m}{s})$

(2 dec + extra)

$= ((17.23\dot{8}\ s)(3.99\ \frac{m}{s})$

4 SF ↑ doesn't count 3 SF

$= 68.8\ m$ (3 SF)

2) A car accelerates uniformly from 1.0 m/s to 15.0 m/s in 5.50 s. What is the rate of acceleration of this car?

$\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$= \dfrac{15.0\ m/s - 1.0\ m/s}{5.50\ s}$ (1 dec, 1 dec)

$= \dfrac{14.0\ m/s}{5.50\ s}$ (3 SF, 3 SF) → we don't add extra when it's "zero" (we didn't round!)

$= 2.55\ m/s^2$ (3 SF)

3) *Area* = (35.98 cm × 34.09 cm) + (107.9 cm × 8.09 cm)

4SF 4SF 4SF 3SF

$(1226.6\,cm^2) + (872.9\,cm^2)$

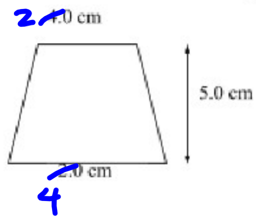
(4SF + extra) (3SF + extra)

$(1226.6\,cm^2) + (872.9\,cm^2)$

0 dec 0 dec

$2100.\ cm^2$ (0 dec)

4) Find the area of this trapezoid.



$$A = \frac{(b+B)h}{2}$$

$$= \frac{(2.0\,cm + 4.0\,cm)(5.0\,cm)}{2}$$

1 dec 1 dec

$$= \frac{(6.0\,cm)(5.0\,cm)}{2}$$

2SF 2SF

$$= 15\,cm^2$$ (2SF)

Uncertainty

Uncertainty is present when a quantity has been measured with an instrument. The uncertainty in the measurement is a result of the uncertainty of the instrument used, or of the skill of the person taking the measurement.

A measurement with uncertainty has 2 parts: the measurement itself and the uncertainty.

Ex:

$$\underbrace{6.85\,cm}_{\text{measurement}} \pm \underbrace{0.05\,cm}_{\text{uncertainty}}$$

There are two ways to express a measurement with uncertainty:

Absolute uncertainty: is expressed in the same units as the measurement itself.

Ex: 5.9 cm ± 0.3 cm

The absolute uncertainty must have the same number of decimal places as the measurementt

Relative uncertainty: is expressed as a percentage of the measurement.

Ex: 5.9 cm ± 5.0 %

$$\text{Relative uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Value of measurement}} \times 100$$

Note: We will express relative uncertainty using ~~one~~ 2 decimal.

There are two ways of determining the uncertainty of a measurement:

1) Using an electronic (digital) device

The uncertainty is usually written on the device itself (or in the manual). If it is indicated, you may assume 1 unit of the smallest increment.

Example: When you mass 2.00 g of NaCl in Chemistry class, you should record it as 2.00 g ± 0.01 g.

2) Using an scaled (analog) device (little lines)

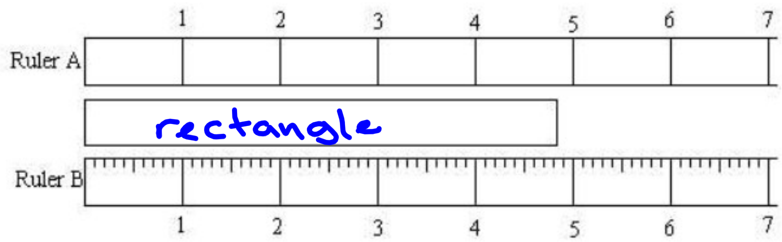
On scaled devices, the uncertainty is equal to "one half of the smallest increment" on the instrument.

A few notes on scaled devices:

- you must **estimate** the last digit
- the measurement and the uncertainty must have the same number of decimals

Examples:

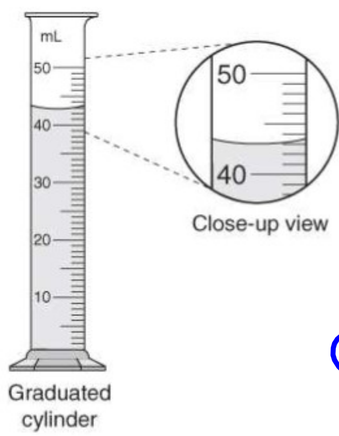
1. Use rulers A and B below to measure the length of the rectangle. Include the absolute uncertainty of the measurement. (Assume the numbers on the rulers are centimeters.)



Ruler A: 4.8 cm ± 0.5 cm   uncert = $\frac{1\text{ cm}}{2}$ = 0.5 cm

Ruler B: 4.85 cm ± 0.05 cm   uncert = $\frac{0.1\text{ cm}}{2}$ = 0.05 cm

2. Read the volume of the liquid in the graduated cylinder. Include the absolute and relative uncertainty. (The volume is in milliliters.)



① Abs

uncert = $\frac{1\text{ mL}}{2}$ = 0.5 mL

$\boxed{43.0\text{ mL} \pm 0.5\text{ mL}}$

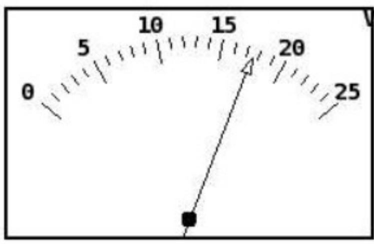
② Rel

Rel uncert = $\frac{0.5\text{ mL}}{43.0\text{ mL}} \times 100$

= 1.16 %

$\boxed{43.0\text{ mL} \pm 1.16\%}$

3. The voltmeter below is graduated in volts. What is the potential difference reading shown by the voltmeter? Include the absolute uncertainty of the measurement.



Voltmeter

uncert $= \frac{1V}{2} = 0.5\ V$

$\boxed{17.7V \pm 0.5\ V}$

4. Express the temperature reading from the thermometer below using both the absolute and relative uncertainty.

smallest possible reading $= 0.1°C$ (uncertainty)

① Abs

uncert $= 0.1°C$

$\boxed{36.8°C \pm 0.1°C}$

② Rel

Rel uncert $= \frac{0.1°C}{36.8°C} \times 100$

$= 0.27\%$

$\boxed{36.8°C \pm 0.27\%}$