Question: How can we model (mathematically) the motion of objects through space and time? (graphically)

Chapter 2: Graphing Motion

Definitions:

Position: location of an object (where it is) (with respect to reference frame)

Symbol: $\vec{d}$ Units: m, km, cm

Displacement: change in position of an object

Symbol: $\Delta\vec{d} = \vec{d}_f - \vec{d}_i$ Units: m, km, cm

($\vec{d}_i$: often "0" when initial position = 0)

Distance: length of path traveled

Symbol: distance Units: m, km, cm

Time interval: time elapsed

Symbol: Δt Units: s, h, min

Velocity (~~instantaneous~~) ~~rate~~ at which position is changing

ex: velocity ~~at~~ 5.0 s

Symbol: $\vec{v}$ Units: m/s, km/h

~~Average~~ Velocity: displacement over a period of time

ex: average velocity ~~from~~ 2.0 s ~~to~~ 6.0 s

Symbol: $\vec{v}_{ave}$ Units: m/s, km/h

~~Instantaneous~~

$$\vec{v}_{ave} = \frac{\text{displacement}}{\Delta t}$$

$$\vec{v}_{ave} = \frac{\Delta\vec{d}}{\Delta t}$$

$$\vec{v}_{ave} = \frac{\vec{d}_f - \vec{d}_i}{\Delta t}$$

Speed: (instantaneous) how fast an object is move at a given moment in time

Symbol: speed Units: m/s , km/h

Average Speed: distance covered over a period of time

Symbol: $speed_{ave} = \frac{distance}{\Delta t}$ Units: m/s , km/h

Acceleration: (Instantaneous) rate at which velocity changes (speed up or slow down)

Symbol: $\vec{a}$ Units: m/s^2

Acceleration: (Average) change in velocity over a period of time

Symbol: $\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t}$ Units: m/s^2

$\vec{a}_{ave} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

Position —rate→ velocity (inst) —rate→ acceleration (average)

Position —change→ displacement → average velocity

Position → distance → average speed

Examples:

1) A cyclist rides 25 km [E] and then 26 km [W] in 2 hours.

W ← −   E → +

   a. What is his average speed?

   $speed_{ave} = \frac{distance}{\Delta t} = \frac{25km + 26km}{2h} = \frac{51km}{2} = 25.5\frac{km}{h}$

   b. What is his average velocity?

   $\vec{v}_{ave} = \frac{displacement}{\Delta t} = \frac{25km - 26km}{2h} = \frac{-1km}{2h} = -0.5\frac{km}{h}$ or $0.5\frac{km}{h}$ [W]

2) A student drives 140 km in 1h15, then takes a 10. min break. He then continues for 240 km in 2.0 hours. What is the average speed for the student on this trip?

   1h15 + 10 + 2h00 = 3h25 = 3.42h

   $speed_{ave} = \frac{distance}{\Delta t} = \frac{140km + 240km}{3.42h} = 111\ km/h$

3) A car accelerates ~~uniformly~~ from 12 m/s to 25 m/s in 4.0 seconds. What is the acceleration of this car? ↳ at constant rate $\vec{a} = \vec{a}_{ave}$

   $\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{25m/s - 12m/s}{4.0s} = 3.25\ m/s^2$

   $\frac{m/s}{s}$ $\frac{m}{s} \times \frac{1}{s}$

4) A cat starts to runs to the right at 1.5 m/s for 5.0 seconds, and then runs in the same direction at a speed of 0.50 m/s for 20. seconds.

   a. What is the instantaneous velocity of the cat 3.0 seconds after it started running? 1.5 m/s

   b. What is the instantaneous velocity of the cat 10 seconds after it started running? 0.50m/s

   c. What is the average velocity of the cat for the 25 seconds?

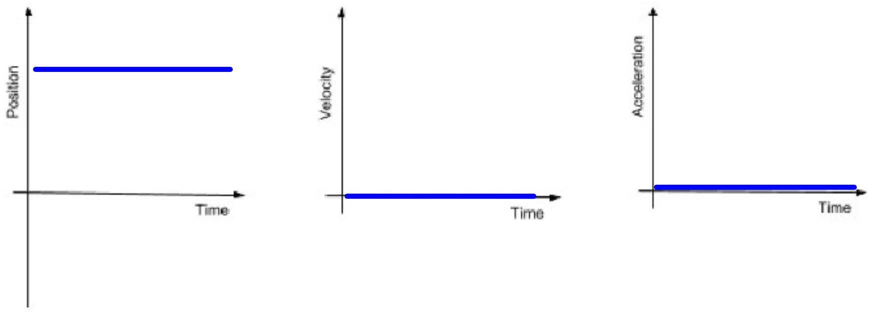
   ← −   → +

   ① 0 → 5s: $5.0s \times \frac{1.5m}{1s} = 7.5m$
   5s → 25s: $20s \times \frac{0.5m}{1s} = 10.m$
   displacement = 17.5m

   ② $\vec{v}_{ave} = \frac{displacement}{\Delta t} = \frac{17.5m}{25s} = +0.7m/s$ (Right)

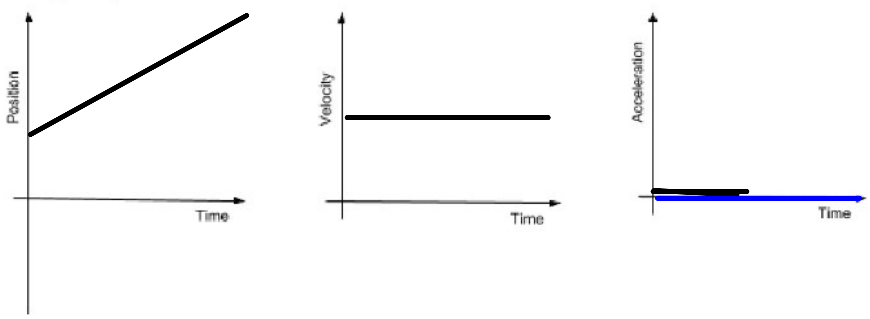
<u>Different Types of Motion</u>

An object at rest (a.k.a. not moving)

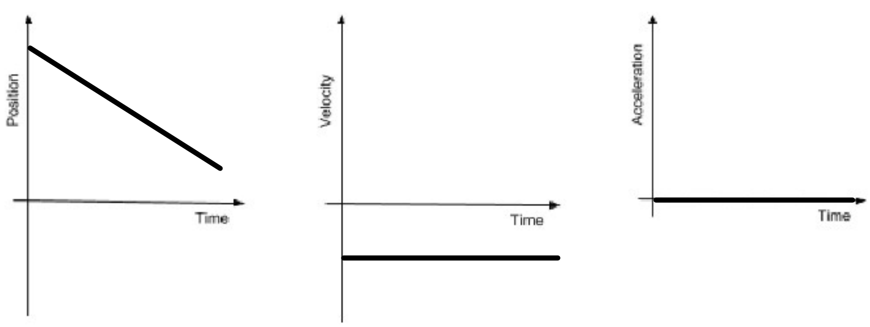


An object moving at a constant velocity (a.k.a. uniform motion)

Moving away

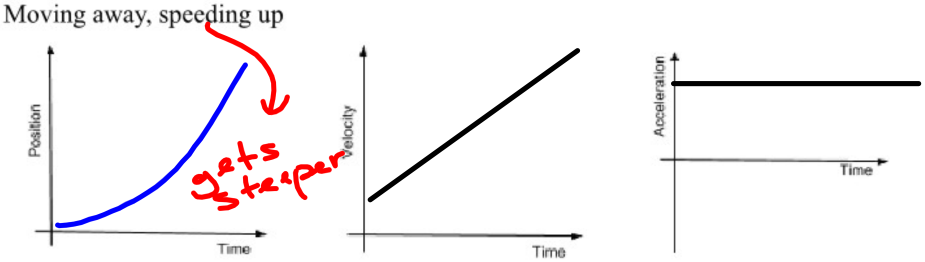


Moving towards

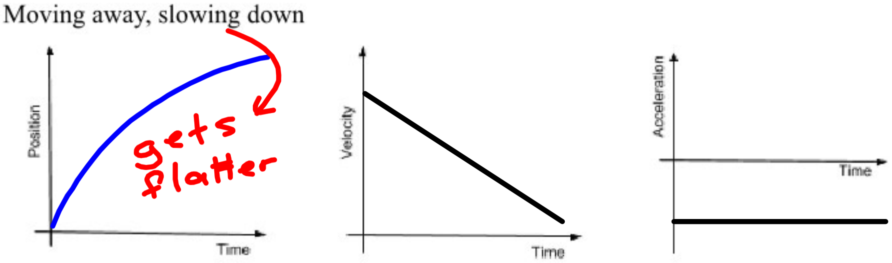


An object undergoing constant acceleration (a.k.a. uniformly accelerated motion)

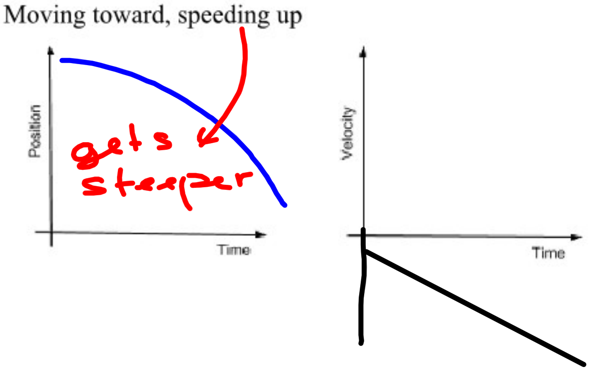
Moving away, speeding up



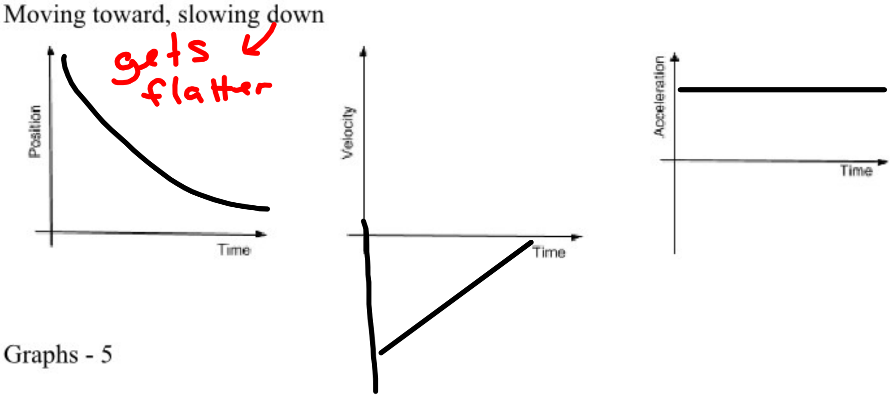
Moving away, slowing down



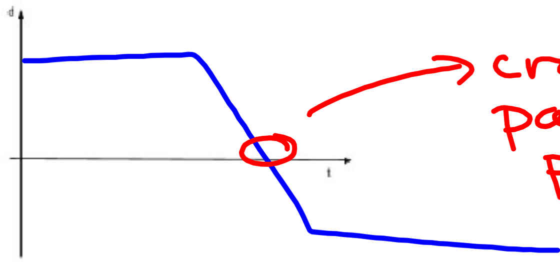
Moving toward, speeding up



Moving toward, slowing down



A closer look at Position vs Time graphs (d-t)



→ cross t-axis = pass the reference point (motion detector)

Information we can get from a d-t graph:

- To find the displacement:

  $\Delta \vec{d} = \vec{d}_f - \vec{d}_i$ (end point, start point)

- To find the distance:

  distance = add magnitudes of individual displacements

- To find the velocity ~~at one point~~ (instantaneous):

  - For a straight line:

    $\vec{v}_{inst}$ = slope (rate of change)

  - For a curve:

    → draw a tangent line at the point (only touches that one point)

    → $\vec{v}_{inst}$ = slope of tangent line

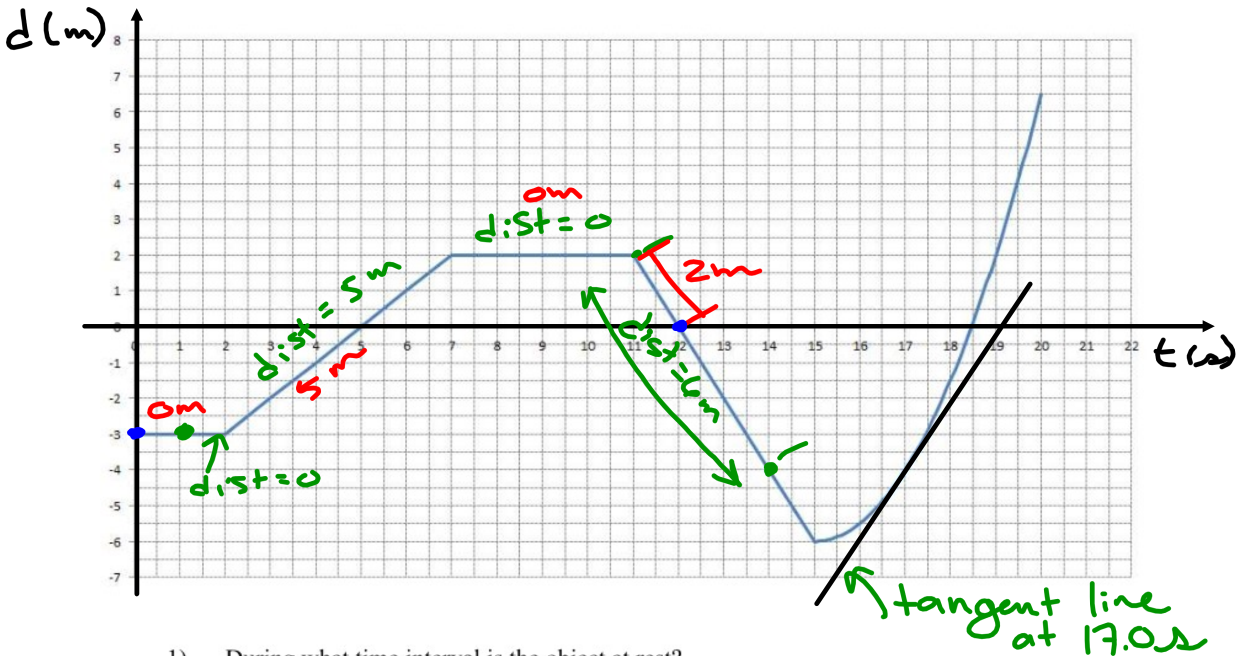
- To find the average velocity:

  $$\vec{v}_{ave} = \frac{\text{displacement}}{\Delta t} = \frac{\vec{d}_f - \vec{d}_i}{\Delta t}$$

- To find the average speed:

  $$\text{Speed}_{ave} = \frac{\text{distance}}{\Delta t}$$ → like above

Examples: The motion of an object is represented by the position-time graph below.



1) During what time interval is the object at rest?

[0,2]s ∪ [7,11]s

2) During what time interval is the object moving backwards? (forward = +)

[11,15]s (neg. slope)

3) During what time interval is the object accelerating?

[15,20]s (curve)

4) What is the displacement of the object from 1 s to 14 s?

$\Delta\vec{d} = \vec{d}_f - \vec{d}_i$

$= -4m - (-3m)$

$= -1m$

5) What is the distance covered from 1 s to 14 s?

distance = 5m + 6m

= 11 m

6) What is the velocity of the object at 5 seconds?

(5,0)
(7,2)

$\vec{v}_{inst} = \text{slope}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2m - 0}{7s - 5s}$

$= \frac{2m}{2s}$

$= 1 m/s$

7) What is the velocity of the object at 12 s?

(12, 0)
(11, 2)

$\vec{v}_{inst} = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2m - 0m}{11s - 12s}$

$= -2\ m/s$

8) What is the velocity of the object at 17 s?

→ draw tangent line at 17.0 s

$\vec{v}_{inst} = \text{slope of tangent}$ (17, −4) (16, −6)

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-4m - (-6m)}{17s - 16s} = 2\ m/s$

9) What is the acceleration of the object at 9.0 s?

$a = 0$ (at rest)

10) What is the average speed of the object from 0 s to 12 s?

$\text{speed}_{ave} = \frac{\text{distance}}{\Delta t}$

$= \frac{5m + 2m}{12s}$

$= 0.58\ m/s$

11) What is the average velocity of the object from 0 s to 12 s?

$\vec{v}_{ave} = \frac{\text{displacement}}{\Delta t} = \frac{\vec{d}_f - \vec{d}_i}{\Delta t}$

$= \frac{0m - (-3m)}{12s} = 0.25\ \frac{m}{s}$

12) What is the average acceleration from 4.0 s to 12.0 s?

slope at 12 s ; slope at 4 s

$\vec{a}_{ave} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

① v_f = slope at 12 s = −2 m/s (#7)

② v_i = slope at 4 s = 1 m/s (same as #6)

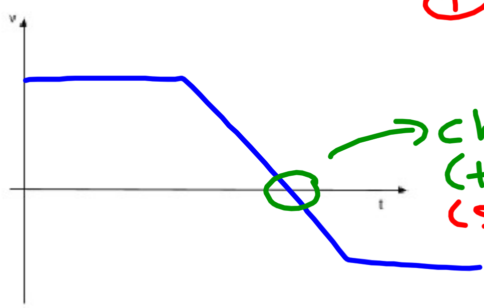
$= \frac{-2m/s - 1m/s}{12s - 4s}$

$= \frac{-3m/s}{8s}$

$= -0.375\ m/s^2$

A closer look at Velocity vs Time graphs (v-t)

⊕ or ⊖ v = opposite directions



→ change in direction (+v to −v) (stop for an instant)

Information we can get from a v-t graph:

- To find the (→ instantaneous) velocity:

  Read the graph! (y-axis)

- To find the acceleration at one point:

  $\vec{a}$ = slope (rate of change)

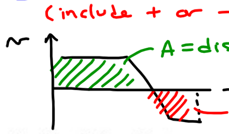
- To find the average acceleration:

  $$\vec{a}_{ave} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

  v_i, v_f = read graph!

- To find the displacement

  $\Delta \vec{d}$ = area between line + axis (include + or −)



- To find the distance:

  distance = area (all values are +)

- To find average velocity:

  $$\vec{v}_{ave} = \frac{\text{displacement}}{\Delta t}$$

  ← areas (with +, −)

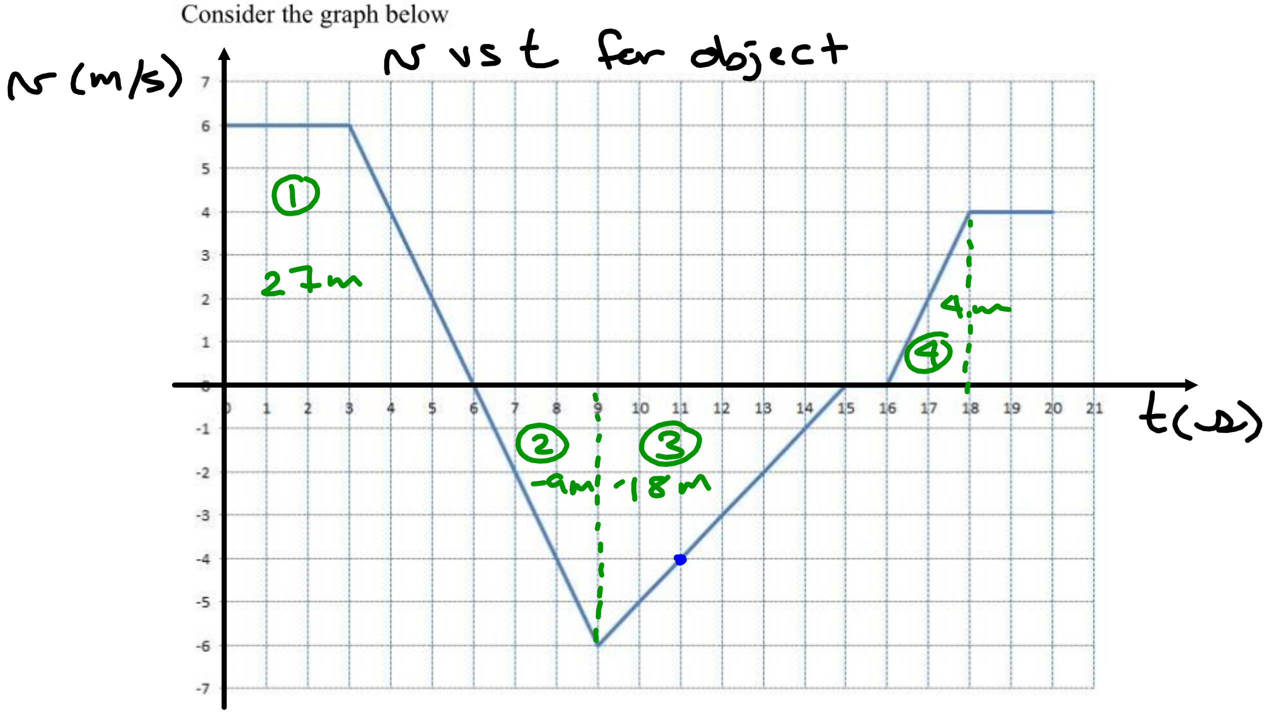
- To find average speed:

  $$\text{Speed}_{ave} = \frac{\text{distance}}{\Delta t}$$

  ← areas (all +)

Example:

Consider the graph below



1) During what time interval(s) is the object at rest?

[15,16] s , at 6.0 s

2) During what time interval(s) is the object speeding up?

[6,9] s ∪ [16,18] s → graph gets ferther from 0

3) During what time interval(s) is the object slowing down?

[3,6] s ∪ [9,15] s → gets closer to 0

4) What is the velocity of the object at 5 s?

2 m/s (read y-axis!)

5) What is the acceleration of the object at 11 s?

(11, -4)
(15, 0)

$\vec{a} = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-4\,m/s)}{15\,s - 11\,s} = \frac{4\,m/s}{4\,s} = 1\,m/s^2$

6) What is the acceleration of the object at 2 s?

a = 0 (constant velocity)

7) What is the average acceleration of the object from 2 s to 13 s?

$\vec{a}_{ave} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$ v_i (2 s) = 6 m/s

$= \frac{-2 m/s - 6 m/s}{11 s}$ v_f (13 s) = -2 m/s

$= -0.73 m/s^2$

8) What is the displacement of the object from 0 s to 9.0 s?

$\Delta \vec{d} = area$

$A_1 = \frac{(B+b)h}{2}$

$= \frac{(6s+3s)(6\frac{m}{s})}{2}$

$= \frac{9s \times 6\frac{m}{s}}{2}$

$= 27 m$

$A_2 = \frac{b \times h}{2}$

$= \frac{3s \times -6\frac{m}{s}}{2}$

$= -9 m$

$\Delta \vec{d} = 27 m + (-9 m)$
$= 18 m$

9) What is the distance covered from 0 s to 9.0 s?

distance = area (all +)
$= 27 m + 9 m$
$= 36 m$

10) What is the average velocity of the object from 0 s to 18 s?

$\vec{v}_{ave} = \frac{disp.}{\Delta t}$

$= \frac{27m - 9m - 18m + 4m}{18 s}$

$= 0.22 m/s$

$A_3 = \frac{b \times h}{2}$

$= \frac{6s \times -6\frac{m}{s}}{2}$

$= -18 m$

$A_4 = \frac{b \times h}{2}$

$= \frac{2s \times 4\frac{m}{s}}{2}$

$= 4 m$

11) What is the average speed from 0 s to 18 s?

$speed_{ave} = \frac{distance}{\Delta t}$

$= \frac{27m + 9m + 18m + 4m}{18 s}$

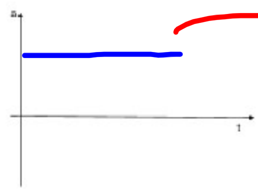
$= 3.2 m/s$

12) Assuming the object stars at zero, where would it be after 3 s?

0 → 3 s $\Delta \vec{d}$ = area
$= b \times h$
$= 3 s \times 6 m/s$
$= 18 m$

The object is at 18 m.

A closer look at Acceleration vs Time graphs (a-t)



→ constant acceleration (speed up or slowing down)

From an a-t graph

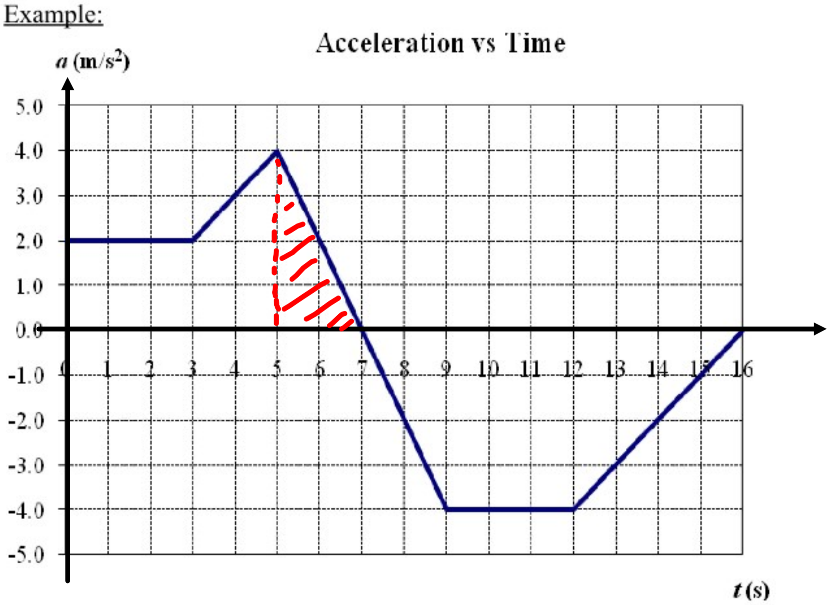
- To find the acceleration:

  read graph (y-axis)

- To find the change in velocity:

  area $= \Delta \vec{v}$

Example:



1) What is the acceleration of the object at 6 s?

$2\ m/s^2$

2) At 5 s, the velocity of the object was 5.0 m/s. What was it velocity at 7 s?

① $\Delta \vec{v} = \text{area} = \dfrac{2\,s \times 4\,\frac{m}{s^2}}{2} = 4\ m/s$

② $\Delta \vec{v} = \vec{v}_f - \vec{v}_i$

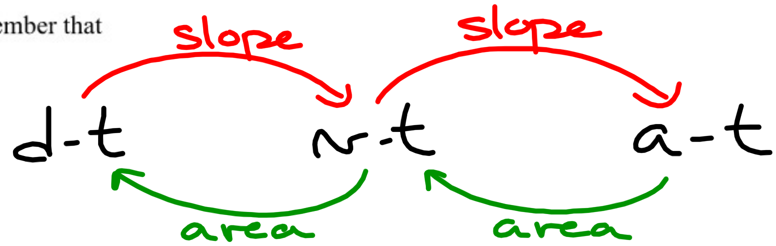
$\vec{v}_f = \vec{v}_i + \Delta \vec{v}$

$= 5.0\ m/s + 4.0\ m/s$

$= 9.0\ m/s$

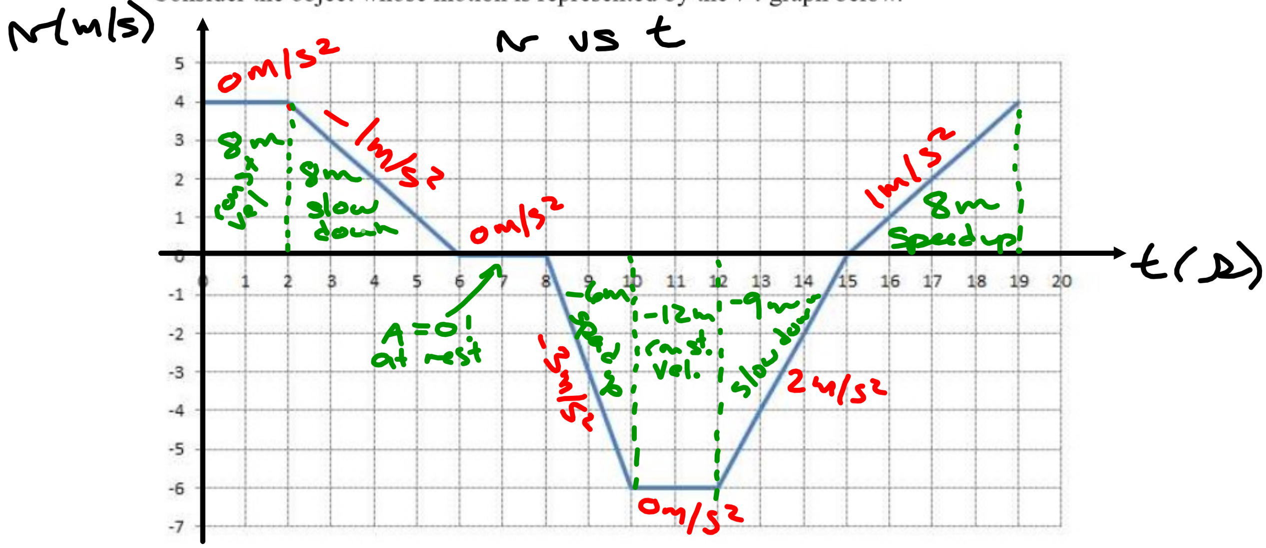
<u>Going back and forth between the different types of graphs</u>

Remember that

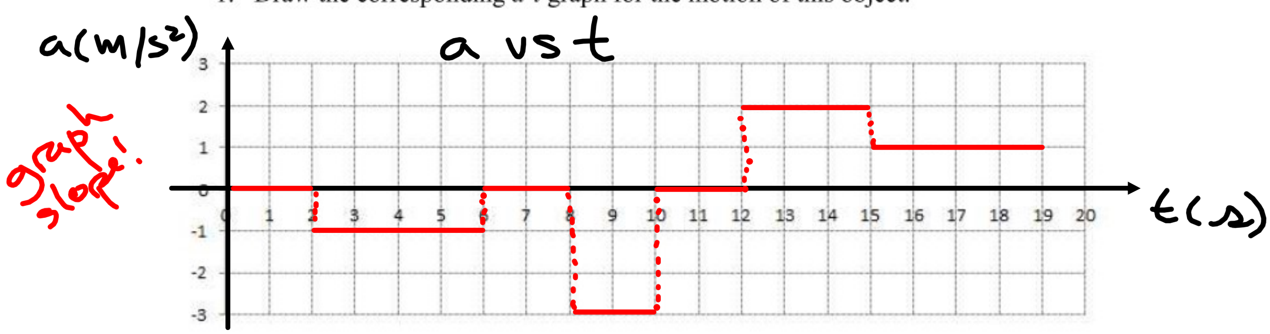


<u>Example:</u>

Consider the object whose motion is represented by the *v*-*t* graph below.



1. Draw the corresponding *a*-*t* graph for the motion of this object.



2. Draw the corresponding *d*-*t* graph for the motion of this object. (Assume the object start at 0.0 m.)