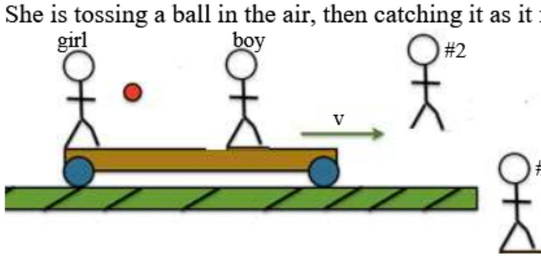
Chapter 5: 2-D Kinematics (Projectile Motion)

Trajectory

The trajectory of an object is its apparent path. The apparent path depends on the location and motion of the observer (i.e. it's all relative!)

Examples:

1. A girl is sitting on a train that moves at a constant velocity as is illustrated below. She is tossing a ball in the air, then catching it as it falls back down.



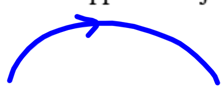
   a. What is the apparent trajectory of the ball for the girl?

   straight up and down

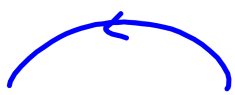
   b. What is the apparent trajectory of the ball for a boy who sits across from her on the train?

   straight up and down

   c. What is the apparent trajectory of the ball for observer 1?



   d. What is the apparent trajectory of the ball for observer 2?



2. Consider a car and a runner on a moving platform.

   a. What is the velocity of the man, relative to the ground?

   10m/s - 5m/s = 5m/s to Right

   b. What is the velocity of the platform, relative to the car?

   20m/s - 10m/s = 10 m/s to Left

   c. What is the velocity of the man, relative to the platform?

   5m/s to left

   (can't feel the moving platform)

So, what is projectile motion?

- The study of the motion of projectiles
- 2-D kinematics deals with objects that move vertically and horizontally at the same time.

Independence of motion

When we study the motion of projectlie, we look at the vertical and horizontal motions seprately.

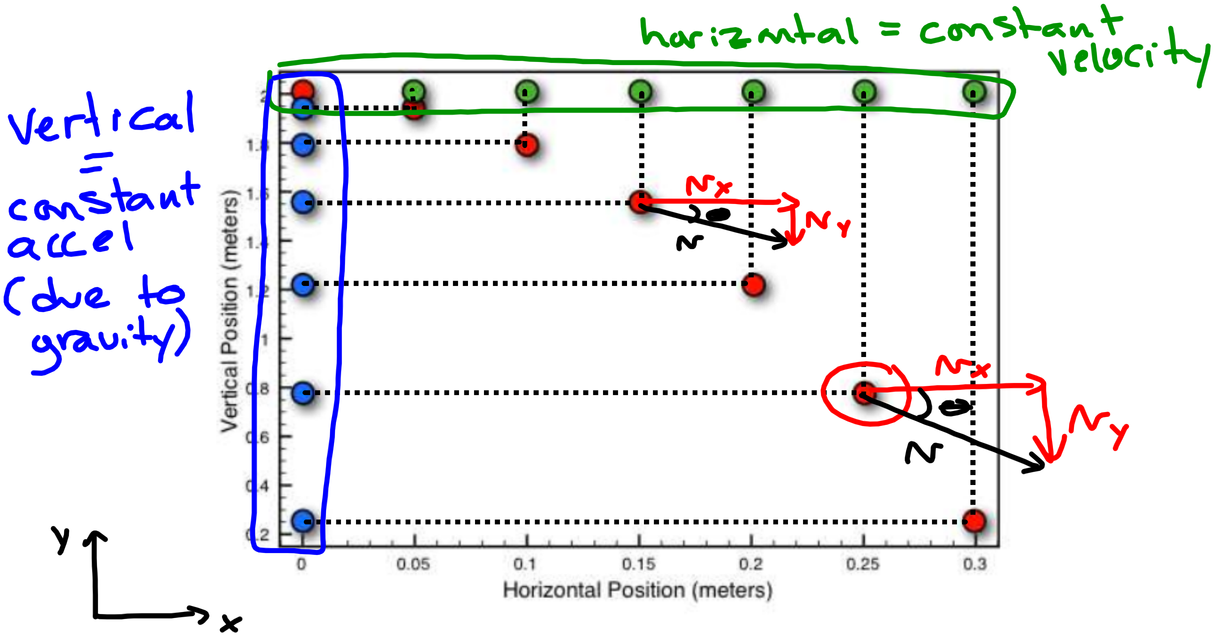
The horizontal and vertical motions do not affect each other; i.e. they are independent.

The link between the horizontal and vertical motions is TIME. Because the projectile travels the horizontal and vertical distances AT THE SAME TIME, Δt is the same for both the horizontal and vertical motions.

In order to solve these problems, we will use the same equations we just learn in the previous chapter.

Case 1: Objects Launched Horizontally (v_i = horizontal)

The diagram below illustrates the motion of a projectile launched horizontally. It is also compared to the motion of an object dropped, and to constant horizontal motion.



x = Horizontal Motion: constant velocity

$$\boxed{\Delta d_x = v_x \Delta t}$$

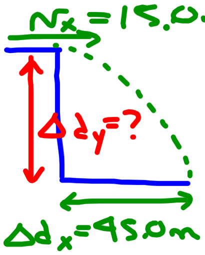
y = Vertical Motion: constant acceleration (due to gravity)

5 same equations, Δd_y, a_y, v_{iy}, v_{fy}, Δt

Examples: * For horizontally launched projectiles $v_{iy} = 0$

$\downarrow +$

1. A car drives off the edge of a cliff at a speed of 15.0 m/s. The car hits the bottom of the cliff 45.0 m from the edge. How high is the cliff?



Hori

$\Delta d_x = 45.0$ m

$v_x = 15.0$ m/s

$\Delta t = ?$

$\Delta d_x = v_x \Delta t$

$\Delta t = \dfrac{\Delta d_x}{v_x}$

$\Delta t = \dfrac{45.0\text{ m}}{15.0\text{ m/s}}$

$\Delta t = 3.00$ s

units: $\cancel{m} \times \dfrac{s}{\cancel{m}}$

Vert

$v_{iy} = 0$

$a_y = 9.8$ m/s²

$\Delta d_y = ?$

$\Delta t = 3.00$ s

$\Delta d_y = \cancel{v_{iy}\Delta t}^{\,0} + \frac{1}{2}a(\Delta t)^2$

$\Delta d_y = \frac{1}{2}(9.8\ \frac{m}{s^2})(3.00\text{ s})^2$

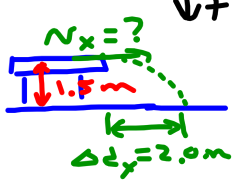
$\Delta d_y = 44.1$ m

ans.

units: $\dfrac{m}{\cancel{s^2}} \times \cancel{s^2}$

$\downarrow +$

2. A marble rolls off the edge of a table 1.5 m high. It hits the ground 2.0 m from the edge of the table. With what speed did the marble roll off the table?



Hori

$\Delta d_x = 2.0$ m

$v_x = ?$

$\Delta t = 0.553$ s

$\Delta d_x = v_x \Delta t$

$v_x = \dfrac{\Delta d_x}{\Delta t}$

$= \dfrac{2.0\text{ m}}{0.553\text{ s}}$

$v_x = 3.6\ \dfrac{m}{s}$

ans

Vert

$v_{iy} = 0$

$\Delta d_y = 1.5$ m

$a = 9.8$ m/s²

$\Delta t = ?$

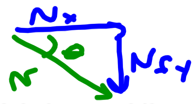
$\Delta d_y = \cancel{v_{iy}\Delta t}^{\,0} + \frac{1}{2}a_y(\Delta t)^2$

$(\Delta t)^2 = \dfrac{2\Delta d_y}{a_y}$

$\Delta t = \sqrt{\dfrac{2(1.5\text{ m})}{9.8\ \frac{m}{s^2}}}$

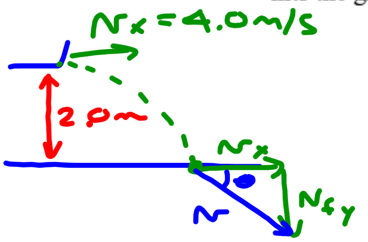
$\Delta t = 0.553$ s

units: $\sqrt{\cancel{m} \times \dfrac{s^2}{\cancel{m}}} = s$



↓+

3. A cat runs off a 2.0 m high balcony while running at a speed of 4.0 m/s. Luckily, the cat lands on its paws and is not injured. What is the velocity of the cat when it hits the ground?



① Find v_{fy}

Vertical only

$v_{iy} = 0$

$\Delta d_y = 2.0\,m$

$a_y = 9.8\,m/s^2$

$v_{fy} = ?$

$v_{fy}^2 = \cancel{v_{iy}^2}^{\,0} + 2a_y\Delta d_y$

$= \sqrt{2(9.8\frac{m}{s^2})(2.0\,m)}$

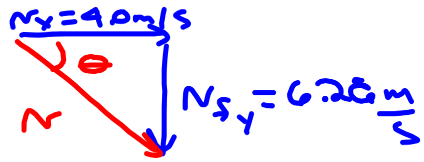
$v_{fy} = 6.26\frac{m}{s}$

(down = ⊕)

units

$\sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$

② Mag + dir for v



$mag = \sqrt{(4.0\frac{m}{s})^2 + (6.26\frac{m}{s})^2}$

$= \sqrt{55.2\frac{m^2}{s^2}}$

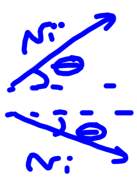
$= 7.4\frac{m}{s}$

$\theta = \tan^{-1}\left(\frac{6.26\,m/s}{4.0\,m/s}\right)$ ← o ← a

$\theta = 57°$

$\boxed{\text{Ans: } 7.4\,m/s,\ 57° \text{ below horizontal}}$

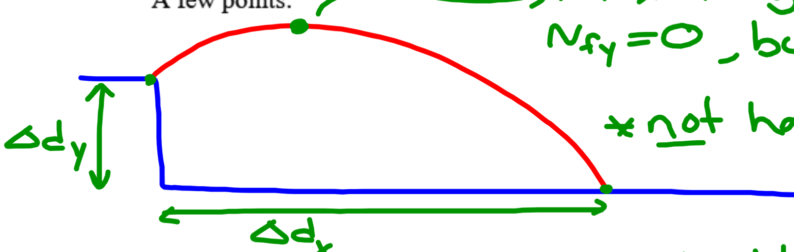
~~Case 2: Objects Launched at an Angle~~



Similar to "objects launched horizontally", except $v_{iy} \neq 0$

When objects are launched at an angle, the initial velocity has BOTH vertical and horizontal components.

A few points:



→ max height

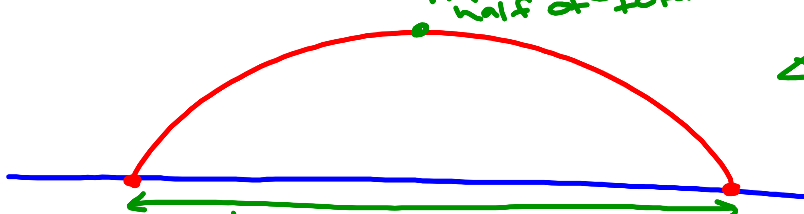
$v_{fy} = 0$, but $v_x \neq 0$

(v_x is constant)

* not half of total time

↑+

↓−



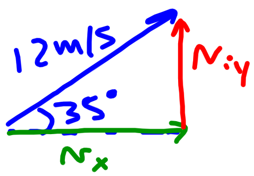
$\Delta d_y = 0$

Δd_x = range

= how far does it go?

Splitting the initial velocity:

Ex: A soccer ball is kicked at a speed of 12 m/s at an angle of 35° above the horizontal. Find the horizontal and vertical components of the initial velocity?



$v_x = 12\frac{m}{s}\cos 35°$

$= 9.83\frac{m}{s}$

$v_{iy} = 12\frac{m}{s}\sin 35°$

$= 6.88\frac{m}{s}$

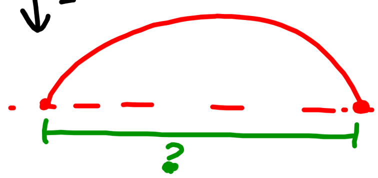
*keep extra SF

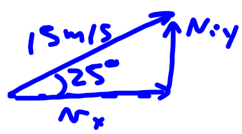
Examples:

↑ +

↓ −

1. Julia kicks a soccer ball that is on the ground, giving it an initial velocity of 15 m/s at an angle of 25 ° above the ground. How far from where she kicked it will the ball hit the ground?





$v_x = 15\frac{m}{s}\cos 25°$

$= 13.6\frac{m}{s}$

$v_{iy} = 15\frac{m}{s}\sin 25°$

$= 6.34\frac{m}{s}$

Hori

$v_x = 13.6\,m/s$

$\Delta d_x = ?$

$\Delta t = 1.29\,s$

$\Delta d_x = v_x \Delta t$

$= (13.6\frac{m}{\cancel{s}})(1.29\cancel{s})$

$= 18\,m$ (2SF)

Vert

$v_{iy} = 6.34\,m/s$

$a = -9.8\,m/s^2$

$\Delta t = ?$

$\Delta d_y = 0$

$v_{fy} = -6.34\,m/s$

OR

$\Delta d_y = 0$

OR

$v_{fy} = 0 \rightarrow$ double Δt

$\cancel{\Delta d_y} = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$0 = v_{iy}\Delta t + \frac{1}{2}a(\Delta t)^2$

$0 = \Delta t\left(v_{iy} + \frac{1}{2}a\Delta t\right)$

$\Delta t = 0$ OR

$0 = v_{iy} + \frac{1}{2}a_y\Delta t$

$-v_{iy} = \frac{1}{2}a\Delta t$

$\Delta t = \frac{-2v_{iy}}{a}$

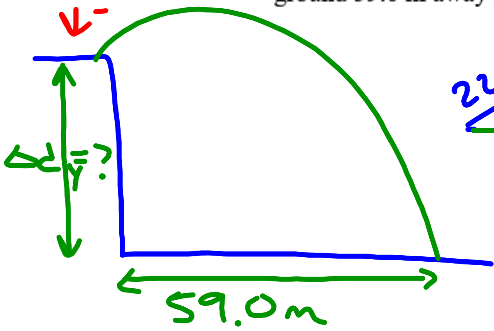
$= \frac{-2(6.34\frac{m}{s})}{-9.8\frac{m}{s^2}}$

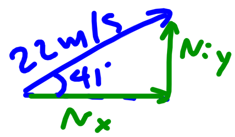
$\frac{\cancel{m}}{\cancel{s}} \times \frac{s^{\cancel{2}}}{\cancel{m}}$

$= 1.29\,s$

2. A ball is kicked from the roof of a building. The ball leaves the kicker's foot with a velocity of 22 m/s at an angle of 41 ° above the horizontal. The ball hits the ground 59.0 m away from the edge of the building. How tall is the building?

↑ +
↓ −





$v_x = 22\frac{m}{s}\cos 41°$
$= 16.6\frac{m}{s}$

$v_{iy} = 22\frac{m}{s}\sin 41°$
$= 14.4\frac{m}{s}$

Horiz

$\Delta d_x = 59.0\,m$
$v_x = 16.6\frac{m}{s}$
$\Delta t = ?$

$\Delta d_x = v_x \Delta t$

$\Delta t = \frac{\Delta d_x}{v_x}$

$= \frac{59.0\,m}{16.6\frac{m}{s}}$ $\quad \cancel{m} \times \frac{s}{\cancel{m}}$

$= 3.55\,s$

Vert

$v_{iy} = 14.4\frac{m}{s}$
$a = -9.8\frac{m}{s^2}$
$\Delta d_y = ?$
$\Delta t = 3.55\,s$

$\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$ $\quad \frac{m}{\cancel{s^2}}\cancel{s^2}$

$= (14.4\frac{m}{\cancel{s}})(3.55\cancel{s}) + \frac{1}{2}(-9.8\frac{m}{s^2})(3.55\,s)^2$

$= 51.1\,m - 61.8\,m$

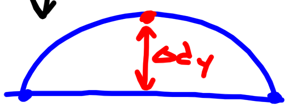
$= -11\,m$

The Building is 11 m tall.

3. A ball is kicked from the ground with an initial speed of 20.0 m/s at an angle of 35.0 ° above the horizontal.

↑+
↓−

a. What is the maximum height reached by the projectile?



20.0 m/s, 35.0°

$v_x = 20.0\,\frac{m}{s}\cos 35°$
$= 16.38\ m/s$

$v_{iy} = 20.0\,\frac{m}{s}\sin 35°$
$= 11.47\ m/s$

Vert

$\Delta d_y = ?$
$v_{iy} = 11.47\ m/s$
$v_{fy} = 0$ (max height)
$a = -9.8\ m/s^2$

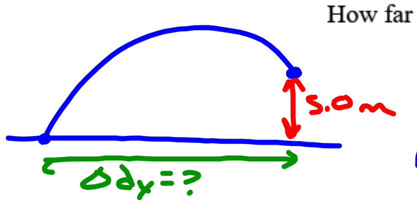
$\cancel{v_f^2}^{\,0} = v_i^2 + 2a\Delta d$

$$\Delta d = \frac{-v_i^2}{2a}$$

$$= \frac{-(11.47\ m/s)^2}{2(-9.8\ m/s^2)}$$

$$\boxed{\Delta d = 6.71\ m}$$

b. On its way down, the ball hits a bird that is flying at an altitude of 5.00 m. How far (horizontally) from the player was the bird?



Vert

$a = -9.8\ m/s^2$
$v_{iy} = 11.47\ m/s$
$\Delta t = ?$
$\Delta d_y = 5.0\ m$

$\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a(\Delta t)^2$

quadratic

$5.0 = (11.47)\Delta t + \frac{1}{2}(-9.8)(\Delta t)^2$

$0 = \underbrace{-4.9}_{a}(\Delta t)^2 + \underbrace{11.47}_{b}\Delta t \underbrace{-5.0}_{c}$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-11.47 \pm \sqrt{(11.47)^2 - 4(-4.9)(-5.0)}}{2(-4.9)}$$

$$= \frac{-11.47 \pm 5.79}{-9.8}$$

+

$$\Delta t = \frac{-11.47 + 5.79}{-9.8}$$
$= 0.58\ s$ way up

$$\Delta t = \frac{-11.47 - 5.79}{-9.8}$$
$= 1.76\ s$ way down

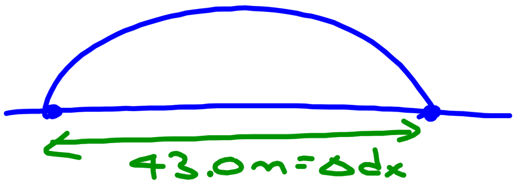
Hori

$\Delta d_x = ?$
$v_x = 16.38\ m/s$
$\Delta t = ?\ 1.76\ s$

$\Delta d_x = v_x\Delta t$
$= (16.38\,\frac{m}{\cancel{s}})(1.76\,\cancel{s})$
$= \underline{28.8\ m}$

A “tough” question

4. A boy kicks a ball from the ground, giving it an angle of 38.0 ° above the ground. The ball hits the ground 43.0 m away. What initial speed did the boy give the ball?



$v_x = v\cos 38°$

$v_x = 0.788v$

$v_{iy} = v\sin 38°$

$v_{iy} = 0.616v$

Hori

$\Delta d_x = 43.0m$

$v_x = 0.788v$

$\Delta t = ?$

$\Delta d_x = v_x \Delta t$

$43.0m = (0.788v)\Delta t$

$\Delta t = \dfrac{43.0m}{0.788v}$

Vert

$\Delta d_y = 0$

$a = -9.8 m/s^2$

$v_{iy} = 0.616v$

$\Delta t = ?$

system

$\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a(\Delta t)^2$

$0 = (0.616v)\Delta t + \frac{1}{2}(-9.8\frac{m}{s^2})\Delta t^2$

$-(0.616v)\cancel{\Delta t} = -4.9\Delta t^{\cancel{2}}$

$-0.616v = -4.9\Delta t$

$0.616v = 4.9\left(\dfrac{43.0}{0.788v}\right)$

$(0.616v)(0.788v) = (4.9)(43.0)$

$0.485v^2 = 210.7$

$v^2 = \dfrac{210.7}{0.485}$

$v = \sqrt{\dfrac{210.7}{0.485}}$

$\boxed{v = 20.8 m/s}$