Chapter 5: 2-D Kinematics (Projectile Motion)

Trajectory

The trajectory of an object is its apparent path. The apparent path depends and the location and motion of the observer (i.e. it's all relative!)

Examples:

1. A soccer ball is kicked up at an angle from the ground. The player kicks the ball straight ahead of him.

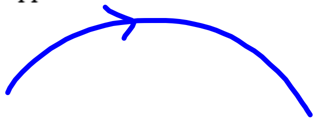
   a. What is the apparent motion of the ball according to the goalie, who stands directly in front of the player.

   straight up then down

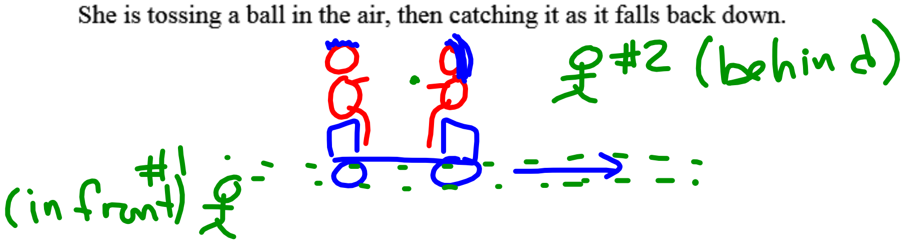
   b. What is the apparent motion of the ball according to a stationary bird (let's pretend) above the field

   c. What is the apparent motion of a ball according to a fan standing on the sideline?

2. A girl is sitting on a train that moves at a constant velocity as is illustrated below. She is tossing a ball in the air, then catching it as it falls back down.

   a. What is the apparent trajectory of the ball for the girl?

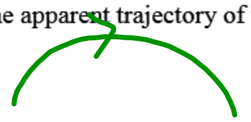
   straight up then down

   b. What is the apparent trajectory of the ball for a boy who sits across from her on the train?

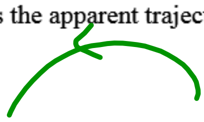
   straight up then down

   same

   c. What is the apparent trajectory of the ball for observer 1?

   d. What is the apparent trajectory of the ball for observer 2?

<u>So, what is projectile motion?</u>

- The study of the motion of projectiles
- 2-D kinematics deals with objects that move vertically and horizontally at the same time.

<u>Independence of motion</u>

When we study the motion of projectlie, we look at the vertical and horizonatl motions seprately.

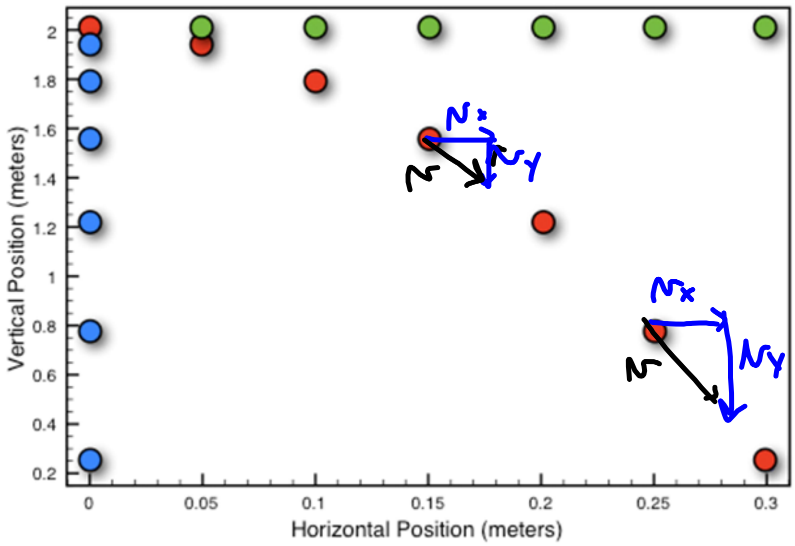
The horizontal and vertical motions do not affect each other; i.e. they are independent.

The link between the horizontal and vertical motions is TIME. Because the projectile travels the horizontal and vertical distances AT THE SAME TIME, Δt is the same for both the horizontal and vertical motions.

In order to solve these problems, we will use the same equations we just learn in the previous chapter.

Case 1: Objects Launched Horizontally

The diagram below illustrates the motion of a projectile launched horizontally. It is also compared to the motion of an object dropped, and to constant horizontal motion.



Horizontal Motion: constant velocity

$\Delta d = v \Delta t$

Vertical Motion: accelerated (due to gravity)

$a = 9.8\ m/s^2$ (down) same 5 equations

* $v_{iy} = 0$ (initial vertical velocity)

Examples:

1. A car drives off the edge of a cliff at a speed of 15.0 m/s. The car hits the bottom of the cliff 45.0 m from the edge. How high is the cliff?

$\downarrow +$

Horizontal

$v_x = 15.0\ m/s$

$\Delta d_x = 45.0\ m$

$\Delta t = ?$

$\Delta d_x = v_x \Delta t$

$\Delta t = \dfrac{\Delta d_x}{v_x} = \dfrac{45.0\ m}{15.0\ \frac{m}{s}} = 3.00\ s$

$m \times \frac{s}{m}$

Vertical

$v_{iy} = 0$

$a = 9.8\ m/s^2$

$\Delta t = 3.00\ s$

$\Delta d = ?$

$\frac{m}{s^2} \times s^2$

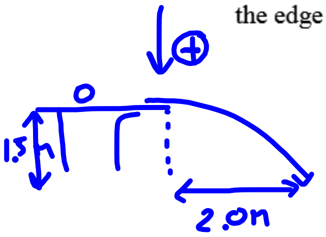
$\Delta d = \cancel{v_i \Delta t}^{\,0} + \frac{1}{2} a (\Delta t)^2$

$= \frac{1}{2}(9.8\ \frac{m}{s^2})(3.00\ s)^2$

$= \boxed{44.1\ m}$

2. A marble rolls off the edge of a table 1.5 m high. It hits the ground 2.0 m from the edge of the table. With what speed did the marble roll off the table?

$\downarrow \oplus$



Horiz

$\Delta d_x = 2.0\ m$

$v_x = ?$

$\Delta t = 0.553\ s$

② $\Delta d_x = v_x \Delta t$

$v_x = \dfrac{\Delta d_x}{\Delta t}$

$= \dfrac{2.0\ m}{0.553\ s}$

$\boxed{v_x = 3.6\ \frac{m}{s}}$

Vertic

$v_{iy} = 0$

$\Delta d_y = 1.5\ m$

$a = 9.8\ \frac{m}{s^2}$

$\Delta t = ?$

① $\Delta d = \cancel{v_i \Delta t}^{\,0} + \frac{1}{2} a (\Delta t)^2$

$(\Delta t)^2 = \dfrac{\Delta d}{\frac{1}{2} a}$

$= \dfrac{(1.5\ m)}{\frac{1}{2}(9.8\ \frac{m}{s^2})}$

$(\Delta t)^2 = 0.30612\ s^2$

$\Delta t = 0.553\ s$

3. A cat runs off a 2.0 m high balcony while running at a speed of 4.0 m/s. Luckily, the cat lands on its paws and is not injured. What is the velocity of the cat when it hits the ground?

$\downarrow (+)$

① Vert

$\Delta d_y = 2.0\,m$

$a = 9.8\,m/s^2$

$v_{iy} = 0$

$v_{fy} = ?$

$v_f^2 = \cancel{v_i^2} + 2a\Delta d$

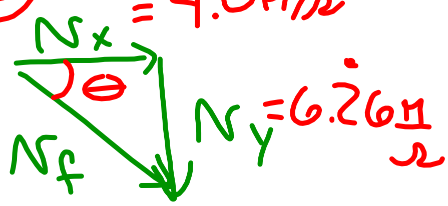
$v_f^2 = 2a\Delta d$

$= 2(9.8\,\frac{m}{s^2})(2.0\,m)$

$v_f^2 = 39.2\,\frac{m^2}{s^2}$

$v_f = 6.\dot{2}\dot{6}\,\frac{m}{s}$

②



$Mag = \sqrt{(4.0\,\frac{m}{s})^2 + (6.26\,\frac{m}{s})^2}$

$= 7.4\,m/s$

$\theta = \tan^{-1}\left(\frac{6.26\,m/s}{4.0\,m/s}\right) = 57°$

$\boxed{\text{Ans: } 7.4\,m/s \text{ @ } 57° \text{ below horiz.}}$

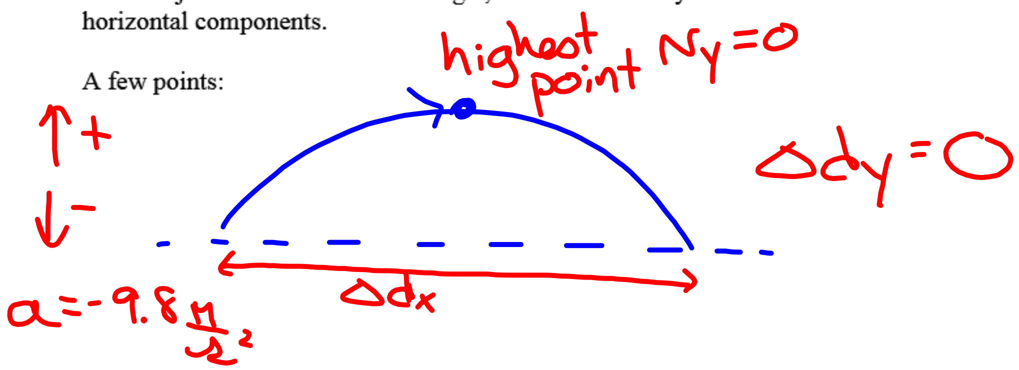
Case 2: Objects Launched at an Angle

Similar to "objects launched ~~at an angle~~ horizontally", except $v_i \neq 0$.

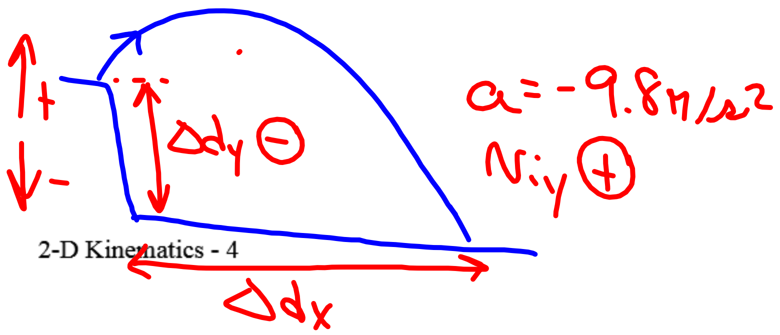
When objects are launched at an angle, the initial velocity has BOTH vertical and horizontal components.

A few points:

①

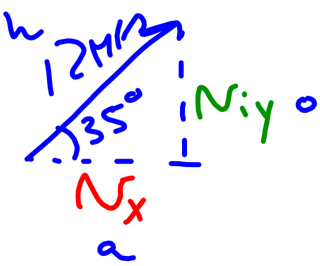


②



Splitting the initial velocity:

Ex: A soccer ball is kicked at a speed of 12 m/s at an angle of 35° above the horizontal. Find the horizontal and vertical components of the initial velocity?



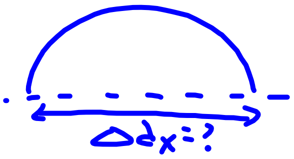
$v_x = 12\ m/s \cos 35°$
$= 9.83\ m/s$

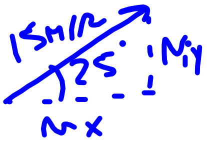
$v_{iy} = 12\ m/s \sin 35°$
$= 6.88\ m/s$

always keep extra SF

Examples:

1. Julia kicks a soccer ball, giving it an initial velocity of 15 m/s at an angle of 25 ° above the ground. How far from where she kicked it will the ball hit the ground?





$v_x = 15\ m/s \cos 25° = 13.6\ m/s$
$v_{iy} = 15\ m/s \sin 25° = 6.34\ m/s$

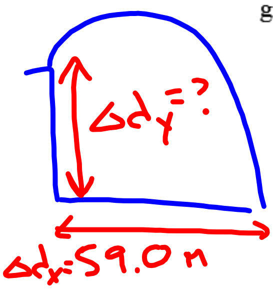
Horiz
$v_x = 13.6\ m/s$
$\Delta d_x = ?$
$\Delta t = 1.29\ s$

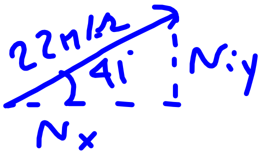
$\Delta d_x = v_x \Delta t$
$= (13.6\ m/s)(1.29\ s)$
$= 18\ m$

Verti
$v_{iy} = 6.34\ m/s$
$a = -9.8\ m/s^2$
$\Delta d_y = 0$
$\Delta t = ?$

$\cancel{\Delta d_y}^{\,0} = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$
$-v_i \cancel{\Delta t} = \frac{1}{2} a (\Delta t)^{\cancel{2}}$
$-v_i = \frac{1}{2} a \Delta t$
$\Delta t = \frac{-v_i}{\frac{1}{2} a}$
$= -\frac{(6.34\ m/s)}{\frac{1}{2}(-9.8\ m/s^2)}$
$= 1.29\ s$

2. A ball is kicked from the roof of a building. The ball leaves the kicker's foot with a velocity of 22 m/s at an angle of 41 ° above the horizontal. The ball hits the ground 59.0 m away from the edge of the building. How tall is the building?





$v_x = 22\text{ m/s}\cos 41° = 16.6\text{ m/s}$

$v_{iy} = 22\text{ m/s}\sin 41° = 14.4\text{ m/s}$

Horiz

$\Delta d_x = 59.0\text{ m}$

$v_x = 16.6\text{ m/s}$

$\Delta t = ?$

$\Delta d_x = v_x \Delta t$

$\Delta t = \dfrac{\Delta d_x}{v_x}$

$= \dfrac{59.0\text{ m}}{16.6\text{ m/s}}$

$= 3.55\text{ s}$

Vert

$\Delta d_y = ?$

$a = -9.8\text{ m/s}^2$

$\Delta t = ?\ 3.55\text{ s}$

$v_{iy} = 14.4\text{ m/s}$

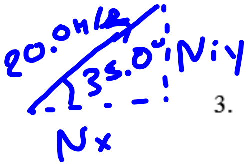
$\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a(\Delta t)^2$

$= (14.4\tfrac{\text{m}}{\text{s}})(3.55\text{ s}) + \frac{1}{2}(-9.8\tfrac{\text{m}}{\text{s}^2})(3.55\text{ s})^2$

$= 51.1\text{ m} - 61.8\text{ m}$

$= -11\text{ m}$

$\Rightarrow$ height of building $= 11\text{ m}$

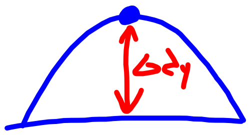


$v_x = 20.0\,m/s \cos 35° = 16.38\,m/s$

$v_{iy} = 20.0\,m/s \sin 35° = 11.47\,m/s$

3. A ball is kicked from the ground with an initial speed of 20.0 m/s at an angle of 35.0 ° above the horizontal.

   a. What is the maximum height reached by the projectile?



<u>Vert</u>

$v_{iy} = 11.47\,m/s$

$a = -9.8\,m/s^2$

$\Delta d_y = ?$

Max height $v_{f_y} = 0$

$\cancel{v_f^2}^{\,0} = v_i^2 + 2a\Delta d$

$-2a\Delta d = v_i^2$

$\Delta d = \dfrac{v_i^2}{-2a}$

$= \dfrac{(11.47\,m/s)^2}{-2(-9.8\,m/s^2)}$

$\boxed{\Delta d = 6.71\,m}$ 3sf

   b. What is the velocity of the projectile, 2.00 s after it was kicked?

<u>Vert</u>

$\Delta t = 2.00\,s$

$v_{iy} = 11.47\,m/s$

$a = -9.8\,m/s^2$

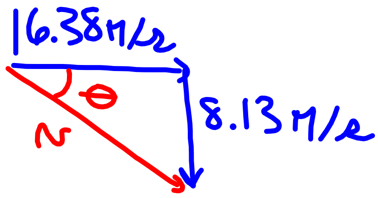
$v_{fy} = ?$

① $v_{f_y} = v_i + a\Delta t$

$= (11.47\,\frac{m}{s}) + (-9.8\,\frac{m}{s^2})(2.00\,s)$

$= 11.47\,\frac{m}{s} - 19.6\,\frac{m}{s}$

$v_{f_y} = -8.13\,m/s$

② 

$Mag = \sqrt{(16.38\,\frac{m}{s})^2 + (8.13\,m/s)^2}$

$= 18.3\,m/s$

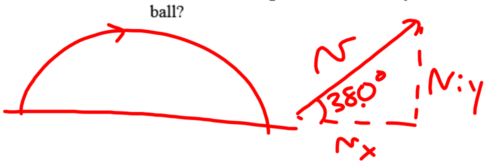
$\theta = \tan^{-1}\left(\dfrac{8.13\,m/s}{16.38\,m/s}\right)$

$= 26.4°$

<u>Ans</u>: 18.3 m/s, 26.4° below horizontal

A "tough" question

4. A boy kicks a ball from the ground, giving it an angle of 38.0 ° above the ground. The ball hits the ground 43.0 m away. What initial speed did the boy give the ball?



$v_x = v\cos 38.0°$
$\quad = 0.7880\,v$

$v_{iy} = v\sin 38.0°$
$\quad = 0.6157\,v$

Hor

$\Delta d_x = 43.0\,\text{m}$

$v_x = 0.7880\,v$

$\Delta t = ?$

Vert

$\Delta d_y = 0$

$a = -9.8\,\text{m/s}^2$

$v_{iy} = 0.6157\,v$

$\Delta t = ?$

Same

$\Delta d_x = v_x \Delta t$

$\Delta t = \dfrac{\Delta d_x}{v_x}$

$\boxed{\Delta t = \dfrac{43.0\,\text{m}}{0.7880\,v}}$

$\cancel{\Delta d_y}^{\,0} = v_{iy}\Delta t + \frac{1}{2}a(\Delta t)^2$

$-v_{iy}\cancel{\Delta t} = \frac{1}{2}a(\Delta t)^{\cancel{2}}$

$-v_{iy} = \frac{1}{2}a\Delta t$

$\Delta t = -\dfrac{v_{iy}}{\frac{1}{2}a}$

$\boxed{\Delta t = -\dfrac{(0.6157\,v)}{\frac{1}{2}(-9.8\,\frac{\text{m}}{\text{s}^2})}}$

$\dfrac{43.0\,\text{m}}{0.7880\,v} = \dfrac{0.6157\,v}{4.9\,\text{m/s}^2}$

$(0.7880\,v)(0.6157\,v) = (43.0\,\text{m})(4.9\,\frac{\text{m}}{\text{s}^2})$

$0.4852\,v^2 = 210.7\,\frac{\text{m}^2}{\text{s}^2}$

$v^2 = \dfrac{210.7\,\frac{\text{m}^2}{\text{s}^2}}{0.4852}$

$v^2 = 434.3\,\text{m}^2/\text{s}^2$

$v = 20.8\,\frac{\text{m}}{\text{s}}$