Chapter 8: Work, Power, Energy

What is work (or work done)?

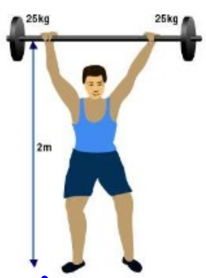
Work is done to transfer energy to a body by means of applying a force (work is energy)

- Symbol: W
  Work is a scalar. → no direction, only magnitude!
- Units of work: Joules (J)

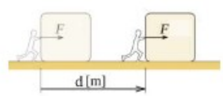
  Note: $1J = 1\frac{kg \cdot m^2}{s^2}$ or $1J = 1N \cdot m$

Mechanical **work is done** when: * force causes the displacement
- the force and the displacement are parallel
- there is an angle (other than 90 °) between the force and the displacement

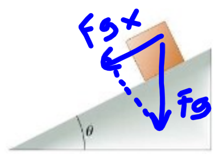
Examples of situations where work IS DONE:



↑F ↑Δd



F →
Δd →

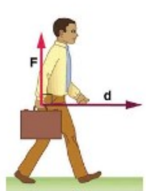


Fgx ↙
Δd ↙

Fax →
Δd →

NO mechanical **work is done** when the force and the displacement are perpendicular.

Examples of situations where NO WORK IS DONE:



F ↑
Δd →
F and Δd are ⊥

Δd = 0

When there is no force...
* doesn't really happen
→ in space?
→ frictionless?

Formula for calculating work:

$$\boxed{W = F \cdot \Delta d}$$

can be a component (F_{gx}, F_{ax}, etc.) of force

$$W = F_{\parallel} \cdot \Delta d$$

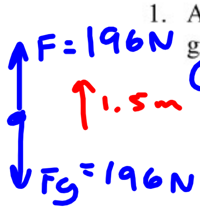
F : in N

Δd : in m

If there is an angle between the force and the displacement, we must find the component of the force that is parallel to the displacement. (Or we can find the component of the displacement parallel to the force.)

Examples:

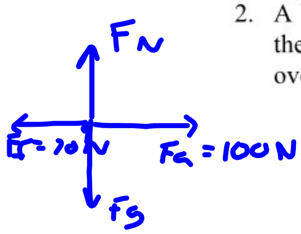
1. A worker lifts a 20 kg box from the floor and places it on a shelf 1.5 m above the ground. How much work does the worker do to accomplish this task?



① $F_g = mg$
$= (20\,kg)(9.8\,m/s^2)$
$= 196\,N$

② $W = F \cdot \Delta d$
$= (196\,N)(1.5\,m)$
$= 294\,J$

2. A boy applies a horizontal force of 100 N (to the right) to a 40 kg box that is on the ground. The boy pushes the box over a frictionless surface for 5.0 m and then over a rough surface where the force of friction is 20 N for 10 m.



a. How much work does the boy do to push the box?

$W_{boy} = F_{boy} \cdot \Delta d$
$= (100\,N)(15\,m)$
$= 1500\,J$

b. How much work does friction do?

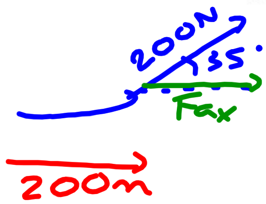
$W_f = F_f \cdot \Delta d$
$= (20\,N)(10\,m)$
$= 200\,J$

* Friction removes energy from the system

c. What is the total work done on the box?

$W_{total} = W_{boy} - W_f$
$= 1500\,J - 200\,J$
$= 1300\,J$

3. A girl pulls a sled with a force of 200 N at an angle of 35 ° above the horizontal. She pulls the sled over a distance of 200 m. How much work does the girl do?



① $F_{ax} = 200N \cos 35°$

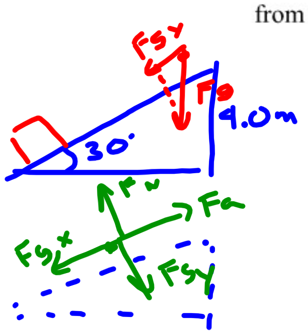
$= 163.8N$

② $W = F_{ax} \cdot \Delta d$

$= (163.8)(200m)$

$= 32760J$

4. A 50 kg box is to be brought to the top of a frictionless incline plane. The incline is set at 30° and is 4.0 m high. Find how much work is needed to get this box from the bottom to the top of the incline.



① $F_g = mg$

$= (50kg)(9.8m/s^2)$

$= 490N$

② $F_{gx} = 490N \sin 30°$

$= 245N = F_a$

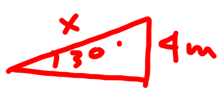
*if friction:

$F_a = F_{gx} + F_f$

③ $W = F_a \cdot \Delta d$

$= (245N)\left(\frac{4m}{\sin 30°}\right)$

$\boxed{W = 1960J}$



$\sin 30° = \frac{4m}{x}$

$x = \frac{4m}{\sin 30°}$

Same answer: $W = (490N)(4m)$

$W = F_g \cdot h$

*On a frictionless ramp, it is the same work to

→ push up the incline

→ lift to top of incline

Mechanical Power

What is power? → "how fast work is done"

Rate at which work can be done

(Rate at which energy is tranfered)

- Symbol: P
  Power is a scalar → magnitude, no direction
- Units of Power: Watts (W)

Note: $1W = 1\frac{J}{s}$

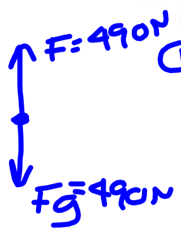
Formula for calculating power:

$$\boxed{P = \frac{W}{\Delta t}}$$

P: power (in W)
W: work (in J)
Δt: time (in s)

Examples:

1. A winch is used to raise a 50 kg box to a height of 10 m above the ground in 20 seconds at a constant velocity. What is the power of the winch?



① $F_g = mg = (50kg)(9.8 m/s^2) = 490 N$

② $W = F \cdot \Delta d = (490N)(10m) = 4900 J$

③ $P = \frac{W}{\Delta t} = \frac{4900J}{20s}$

$P = 245 W$

2. A girl pushes a box on the floor at a constant velocity of 1.5 m/s. She exerts a horizontal force of 100 N.[redacted]. What is the power generated by the girl?

① $P = \frac{W}{\Delta t}$

$P = \frac{F \cdot \Delta d}{\Delta t}$

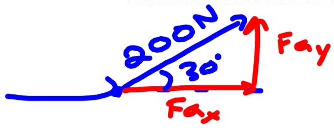
$P = F \cdot \left(\frac{\Delta d}{\Delta t}\right)$ ← v for constant velocity

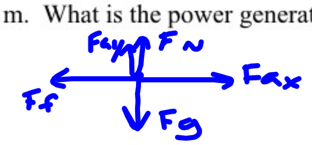
$P = F \cdot v$

$P = F \cdot v = (100 N)(1.5 m/s)$

$P = 150 W$

3. A boy pulls a 20 kg sled using a force of 200 N at an angle of 30º above the horizontal. Friction provides a force of 100 N. The sled starts from rest, and covers a distance of 8.0 m. What is the power generated by the boy?





① $F_{ax} = 200\,N\cos 30°$
$= 173.2\,N$
$F_{ay} = 200\,N\sin 30°$
$= 100\,N$

② $W_{boy} = F_{ax\,boy} \cdot \Delta d$
$= (173.2\,N)(8.0\,m)$
$= 1386\,J$

*To find Δt, need a.

③ $F_{net} = F_{ax} - F_f$
$= 173.2\,N - 100\,N$
$= 73.2\,N$

④ $F_{net} = ma$
$a = \frac{F_{net}}{m}$
$= \frac{73.2\,N}{20\,kg}$
$= 3.66\,m/s^2$

⑤ $\Delta d = 8.0\,m$
$a = 3.66\,m/s^2$
$v_i = 0$
$\Delta t = ?$

$\Delta d = \cancel{v_i \Delta t}^{\,0} + \frac{1}{2}a(\Delta t)^2$
$(\Delta t)^2 = \frac{2\Delta d}{a}$
$\Delta t = \sqrt{\frac{2(8.0\,m)}{3.66\,m/s^2}}$
$\Delta t = 2.09\,s$

⑥ $P = \frac{W}{\Delta t}$
$= \frac{1386\,J}{2.09\,s}$
$\boxed{P = 663\,W}$

Mechanical Energy

Mechanical energy is composed of kinetic energy and gravitational potential energy.

Gravitational Potential Energy

Potential energy is the amount of energy associated with the position (height) of an object with respect to a reference point.

$\Rightarrow$ we can choose $h = 0$ to be anything

- Symbol: E_p
  Energy is a scalar.
- Units of Energy: Joules (J)

  Note: $1J = 1\dfrac{kgm^2}{s^2}$

- Formula: $\boxed{E_p = mgh}$

E_p : potential Energy (J)
m : mass (kg)
g : grav. const (m/s^2)
h : height (from "0") (m)

Kinetic Energy

Kinetic energy is the amount of energy associated with the motion of an object.

- Symbol: E_k
  Energy is a scalar.
- Units of Energy: Joules (J)

  Note: $1J = 1\dfrac{kgm^2}{s^2}$

- Formula: $\boxed{E_k = \dfrac{1}{2}mv^2}$

E_k = kinetic energy (J)
m : mass (kg)
v : velocity (m/s)

Examples:

1. A 2500 kg car travels a speed of 60 km/h. How much kinetic energy does this car have?

① $60\dfrac{\cancel{km}}{\cancel{h}} \times \dfrac{1000\,m}{1\,\cancel{km}} \times \dfrac{1\,\cancel{h}}{3600\,s} = 16.67\,m/s$

② $E_k = \dfrac{1}{2}mv^2$

$= \dfrac{1}{2}(2500\,kg)(16.67\,m/s)^2$

$= 347361\,J$

$= 347\,kJ$

$\to 0.5\,kg$

2. A 500 g apple is in a tree. The apple has 12.25 J of potential energy relative to the ground. How high above the ground is the apple located?

ground is h=0

$$E_p = mgh$$

$$h = \frac{E_p}{mg} = \frac{12.25\,J}{(0.5\,kg)(9.8\,m/s^2)}$$

$$\boxed{h = 2.5\,m}$$

units

$$\frac{J}{kg\cdot\frac{m}{s^2}} = \cancel{kg}\cdot\frac{m^{\cancel{2}}}{\cancel{s^2}} \times \frac{1}{\cancel{kg}} \times \frac{\cancel{s^2}}{\cancel{m}}$$

Conservation of Mechanical Energy

$\to$ no external force (friction, Fa...)

In a close (isolated) system, the total mechanical energy of system is constant.

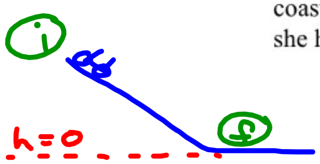
This mean: $Em_i = Em_f$

~~$Ep_i + Ek_i = Ep_f + Ek_f$~~

i = initial
f = final

Examples:

1. A cyclist and her bicycle have a combined mass of 65 kg. She starts from rest and coasts down the hill without pedaling. When she reaches the bottom of the hill, she has a speed of 12 m/s. What is the height of the hill? (Friction is negligible.)



$$① \; \cancel{E_{k_i}}^{\,0} + E_{p_i} = E_{k_f} + \cancel{E_{p_f}}^{\,0}$$

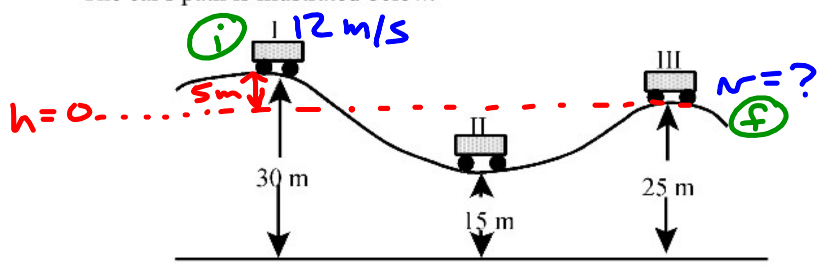
$$mgh_i = \frac{1}{2} m v_f^2$$

$$h_i = \frac{\frac{1}{2}\cancel{m} v_f^2}{\cancel{m} g} = \frac{\frac{1}{2}(12\,m/s)^2}{9.8\,m/s^2}$$

$$\boxed{h_i = 7.35\,m}$$

2. A roller coaster car passes through point I at a speed of 12 m/s and then keeps going, passing through points II and III.

The car's path is illustrated below.



Frictional forces are negligible. What is the speed of the cart at point III?

$$① \; E_{k_i} + E_{p_i} = E_{k_f} + \cancel{E_{p_f}}^{\,0}$$

$$\frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_f^2$$

$$v_f^2 = \frac{\frac{1}{2}v_i^2 + gh}{\frac{1}{2}}$$

$$v_f = \sqrt{\frac{\frac{1}{2}(12\,m/s)^2 + (9.8\,m/s^2)(5m)}{1/2}}$$

$$v_f = 15.6\,m/s$$

Work done by friction forces

When there is friction on the system, total energy is still conserved. Friction does work to remove energy from the system.

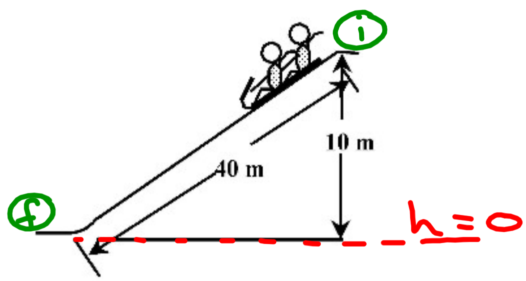
So this means: $Ep_i + Ek_i - W_f = Ep_f + Ek_f$

* can't cancel mass

$$\boxed{W_f = F_f \cdot \Delta d}$$

Example:

1. Two ~~persons~~ kids slide (from rest) down a snow- covered hill in a sled from a height of 10 m. The force of friction on the sledders is 200 N. The total mass of the ~~persons~~ kids and sled is 100 kg. The slope of the hill is 40 m long. Calculate the speed of the sled and its occupants at the bottom of the hill.



① $Ep_i + \cancel{Ek_i}^{\,0} - W_f = \cancel{Ep_f}^{\,0} + Ek_f$

$Ep_i - W_f = Ek_f$

$mgh - F_f \cdot \Delta d = \frac{1}{2} m v^2$

$v^2 = \dfrac{mgh - F_f \cdot \Delta d}{\frac{1}{2} m}$

$v = \sqrt{\dfrac{(100\text{kg})(9.8\text{m/s}^2)(10\text{m}) - (200\text{N})(40\text{m})}{\frac{1}{2}(100\text{kg})}}$

$$\boxed{v = 6 \text{ m/s}}$$

Work done by external (applied) forces

We saw that when a force of friction does work, it removes energy from the system.

When a force is applied to a system, it can ADD energy to the system.

So the equation would look something like:

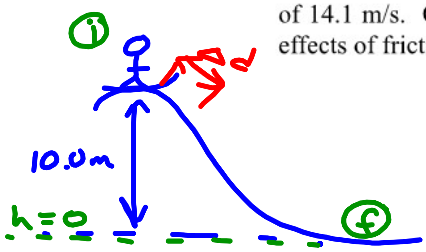
$Ep_i + Ek_i - W_f + W_{fa} = Ep_f + Ek_f$

*can't cancel mass

$$\boxed{W_a = F_a \cdot \Delta d}$$

Example:

2. A 65.0 kg skier is at rest at the top of a hill 10.0 m high. She pushes with her poles with a force of 50.0 N, to give herself an initial speed. She then coasts down the rest of the hill. When she reaches the bottom of the hill, she has a speed of 14.1 m/s. Over what distance did the skier push with her poles? Disregard the effects of friction.



1. $Ep_i + \cancel{Ek_i}^{0} - \cancel{W_f}^{0} + W_a = \cancel{Ep_f}^{0} + Ek_f$

$Ep_i + W_a = Ek_f$

$W_a = Ek_f - Ep_i$

$= \frac{1}{2}mv^2 - mgh$

$= \frac{1}{2}(65.0\,kg)(14.1\,m/s)^2 - (65.0\,kg)(9.8\,m/s^2)(10\,m)$

$= 6461.3\,J - 6370\,J$

$W_a = 91.3\,J$

2. $W_a = F_a \cdot \Delta d$

$\Delta d = \frac{W_a}{F_a}$

$= \frac{91.3\,J}{50.0\,N}$

$$\boxed{\Delta d = 1.8\,m}$$

Elastic Energy (Energy in a Spring)

When a spring is stretched or compressed, it stores elastic potential energy. When the spring is released, the extremity of the spring moves, transforming the elastic potential energy into kinetic energy.

E_e in equation:
*if spring ~~starts~~ stretche/compressed → "i" side
*if springs ~~ends~~ stretch/compressed → "f" side

- Symbol: E_e
  Energy stored in a spring is a scalar.

- Units of Energy: Joules (J)

Note: $1J = 1\frac{kgm^2}{s^2}$

- Formula: $\cancel{E_e = \frac{1}{2}kx^2}$ $\boxed{E_e = \frac{1}{2}k(\Delta x)^2}$

k: spring constant (N/m)
Δx: deformation (m)

Examples:

1. In order to compress a spring by 40 cm, 48 J of work must be done. What is the spring constant of this spring?

$$E_e = \frac{1}{2}k(\Delta x)^2$$
$$k = \frac{E_e}{\frac{1}{2}(\Delta x)^2}$$
$$= \frac{48J}{\frac{1}{2}(0.4m)^2}$$
$$k = 600N/m$$

units

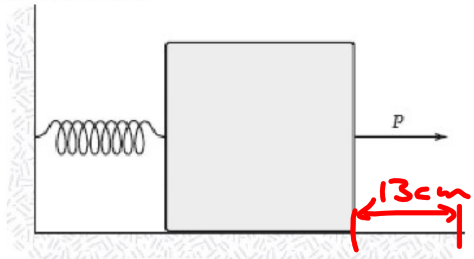
$$\frac{J}{m^2} \rightarrow \frac{N\cancel{m}}{m^{\cancel{2}}}$$

2. A dart of mass 0.100 kg is pressed against a spring of a toy dart gun. The spring has a spring constant k = 250 N/m, and it is compressed by a distance of 0.06m. If the dart detaches from the spring once the spring reaches it's equilibrium position, what is the speed of the dart as it leaves the spring?

*spring starts compressed

$$\cancel{E_{pi}}^0 + \cancel{E_{ki}}^0 - \cancel{W_f}^0 + \cancel{W_a}^0 + E_e = \cancel{E_{pf}}^0 + E_{kf}$$
$$E_e = E_{kf}$$
$$\cancel{\frac{1}{2}}k(\Delta x)^2 = \cancel{\frac{1}{2}}mv^2$$
$$v^2 = \frac{k(\Delta x)^2}{m}$$
$$v = \sqrt{\frac{(250N/m)(0.06m)^2}{(0.100kg)}}$$
$$v = 3m/s$$

3. A 10-kg block on a horizontal frictionless surface is attached to a light spring of constant 800 N/m. The block is initially at rest at the spring's equilibrium position when a force (magnitude P = 80 N) acting parallel to the surface is applied to the block, as shown. What is the speed of the block when it is 13 cm from its equilibrium position?



*spring stretch at ~~end~~

① $\cancel{E_{pi}}^{0} + \cancel{E_{ki}}^{0} - \cancel{W_f}^{0} + W_a = \cancel{E_{pf}}^{0} + E_{kf} + E_e$

$W_a = E_{kf} + E_e$

$E_{kf} = W_a - E_e$

$= F_a \cdot \Delta d - \frac{1}{2}k(\Delta x)^2$

$= (80\,N)(0.13\,m) - \frac{1}{2}(800\,N/m)(0.13\,m)^2$

$= 10.4\,J - 6.76\,J$

$E_{kf} = 3.64\,J$

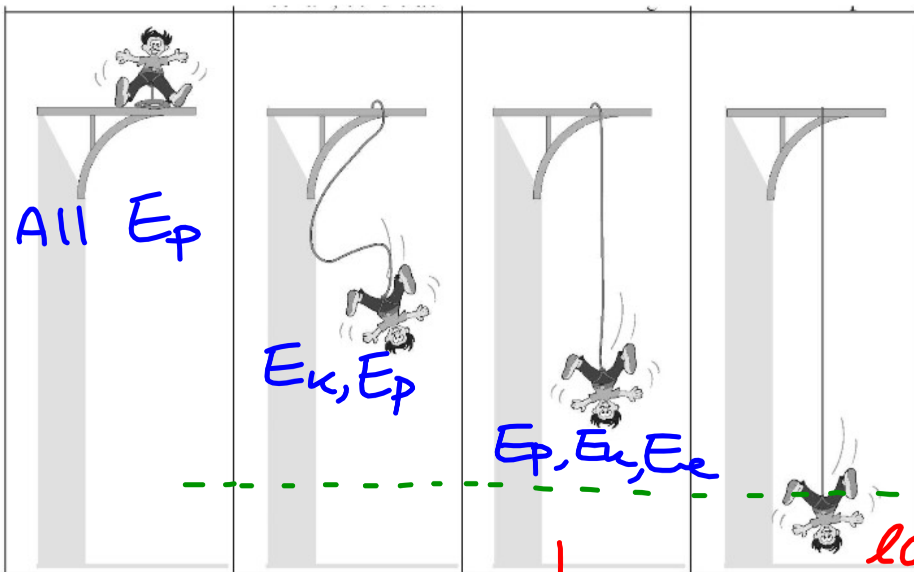
② $E_k = \frac{1}{2}mv^2$

$v^2 = \frac{2E_k}{m}$

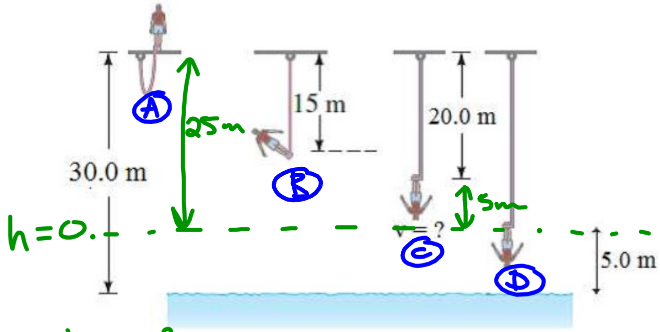
$v = \sqrt{\frac{2(3.64\,J)}{(10\,kg)}}$

$v = 0.85\,m/s$

A little something about the bungee (or mass that fall while attached to a spring)…



Example: A 60.0 kg bungee jumper jumps from a platform that 30.0 m above the ground. The unstretched elastic is 15 m long. She reaches her lowest point when she is 5.0 m above the ground. How fast is she moving when she is 20.0 m below the platform?



① From $\overset{i}{A} \rightarrow \overset{f}{D}$

$$E_{p_i} = E_e$$

$$mgh = \frac{1}{2}k(\Delta x)^2$$

$$k = \frac{mgh}{\frac{1}{2}(\Delta x)^2}$$

$\Delta x = 10m$
$= (25m - 15m)$

$$= \frac{(60.0kg)(9.8m/s^2)(25m)}{\frac{1}{2}(10m)^2}$$

$$k = 294\ N/m$$

② From ^{i}A to C^{f}

$$E_{p_i} = E_{k_f} + E_{p_f} + E_e$$

$$E_{k_f} = E_{p_i} - E_{p_f} - E_e$$

$$= mgh_i - mgh_f - \frac{1}{2}k(\Delta x)^2$$

$$= (60.0kg)(9.8\tfrac{m}{s^2})(25m) - (60kg)(9.8\tfrac{m}{s^2})(5m) - \frac{1}{2}(294\tfrac{N}{m})(5m)^2$$

$$E_{k_f} = 8085\ J$$

③ $E_{k_f} = \frac{1}{2}mv^2$

$$v^2 = \frac{2E_k}{m}$$

$$v = \sqrt{\frac{2(8085J)}{60.0kg}}$$

$$= 16.4\ m/s$$

Work-Energy Theorem

When work is done on an object, it changes the kinetic energy of the object.

So the total work done on an object corresponds to the change in kinetic energy of that object.

$$\cancel{E_{p_i}}^{\,0} + E_{k_i} - W_f + W_a = \cancel{E_{p_f}}^{\,0} + E_{k_f}$$

$$W_a - W_f = E_{k_f} - E_{k_i}$$

$$W_{total} = \Delta E_k$$

This means: $W_{total} = \Delta E_K$

$$\boxed{W_{total} = Ek_f - Ek_i}$$

Examples:

1. A 2000 kg car going at speed of 25 m/s comes to a stop over a distance of 70 m. What is the force applied to the car by the brakes?

$$\cancel{W_a}^{\,0} - W_f = \cancel{E_{k_f}}^{\,0} - E_{k_i}$$

$$W_f = E_{k_i}$$

$$F_f \cdot \Delta d = \frac{1}{2} m v^2$$

$$F_f = \frac{\frac{1}{2} m v^2}{\Delta d}$$

$$= \frac{\frac{1}{2}(2000kg)(25m/s)^2}{70m}$$

$$\boxed{F_f = 8929 N}$$

2. A wagon starting from rest is pulled with a force of 25 N over a distance of 5.0 m. Friction exerts a force of 10 N. What is the final kinetic energy of the wagon?

$$W_a - W_f = E_{k_f} - \cancel{E_{k_i}}^{\,0}$$

$$E_{k_f} = F_a \cdot \Delta d - F_f \cdot \Delta d$$

*not always same

$$= (25N)(5m) - (10N)(5m)$$

$$\boxed{E_{k_f} = 75 J}$$