Chapter 7: Forces and Newton's Laws (Part II)

What now?

We will now look at situations where forces are being applied in various directions (not just parallel and perpendicular to the motion).

We will look at horizontal and vertical forces separately.

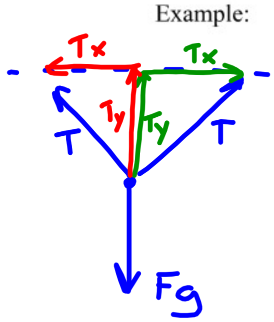
We will split forces into components that are either
- Parallel to motion "x"
- Perpendicular to motion "y"

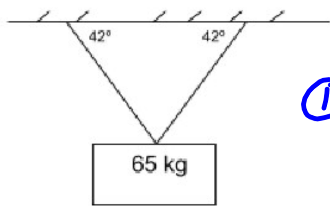
Note: When and object is at rest, $F_{net} = 0$ horizontally and vertically.

Case 1: Static Equilibrium → no motion (equilibrium → $a = 0$)

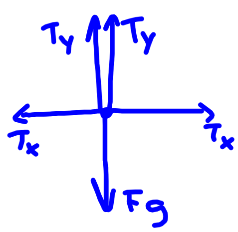
Symmetrical situations: → same Tension in each string

Example: A sign is supported by two strings, as illustrated below. What is the tension in the strings?





① $F_g = mg$
$= (65\,kg)(9.8\,m/s^2)$
$= 637\,N$



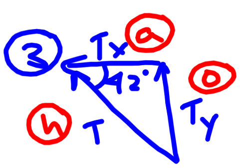
② $T_y + T_y = F_g$ (equilibrium)

$2T_y = F_g$

$T_y = \frac{F_g}{2}$

$= \frac{637\,N}{2}$

$T_y = 318.5\,N$



$\sin 42° = \frac{T_y}{T}$

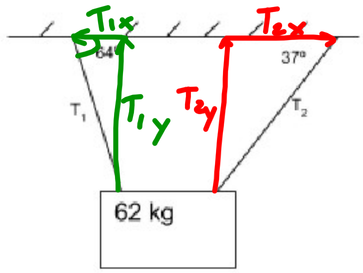
$T = \frac{T_y}{\sin 42°}$

$T = \frac{318.5\,N}{\sin 42°}$

$\boxed{T = 476\,N}$

Non-Symmetrical situations: $\longrightarrow T_2 \neq T_1$

Example: A sign is supported by two strings, as illustrated below. What is the tension in each strings?

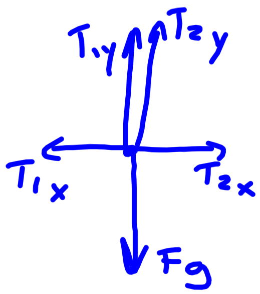


$T_{1x} = T_1 \cos 64°$

$T_{1y} = T_1 \sin 64°$

$T_{2x} = T_2 \cos 37°$

$T_{2y} = T_2 \sin 37°$



$F_g = mg$
$= (62\,kg)(9.8\,\frac{m}{s^2})$
$= 607.6\,N$

① $T_{1x} = T_{2x}$

$T_1 \cos 64° = T_2 \cos 37°$

$T_1 = \frac{T_2 \cos 37°}{\cos 64°}$

$\boxed{T_1 = 1.822\,T_2}$

② $T_{1y} + T_{2y} = F_g$

$T_1 \sin 64° + T_2 \sin 37° = 607.6\,N$

$(1.822\,T_2)\sin 64° + T_2 \sin 37° = 607.6\,N$

$1.638\,T_2 + 0.602\,T_2 = 607.6\,N$

$2.24\,T_2 = 607.6\,N$

$T_2 = \frac{607.6\,N}{2.24}$

$\boxed{T_2 = 271\,N}$

③ $T_1 = 1.822\,T_2$

$= 1.822(271\,N)$

$\boxed{T_1 = 494\,N}$

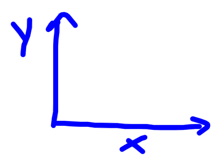
~~Case 2~~: Pulling at an angle pushing

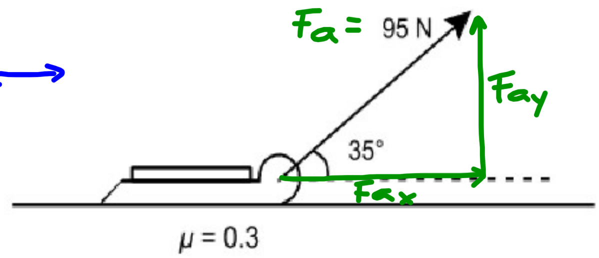
An object is being pulled or pushed, but the force is being applied at an angle.

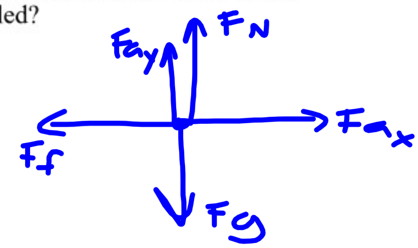
Look at the ~~horizontal~~ x (usually ~~parallel~~ to motion) and ~~vertical~~ y (usually ~~perpendicular~~ to motion) components separately.

Remember that in this case $F_N \neq F_g$

Example: A 15 kg sled is being pulled with a force of 95 N at an angle of 35° to the horizontal. What is the acceleration of the sled?







① $F_g = mg$
$= (15\,kg)(9.8\,m/s^2)$
$= 147\,N$

② $F_{a_x} = 95\,N\cos 35°$
$= 77.8\,N$
$F_{a_y} = 95\,N\sin 35°$
$= 54.5\,N$

③ $\cancel{F_{a_y} + F_N = F_g}$ $(F_g \neq F_N)$
$F_N = F_g - F_{ay}$
$= 147\,N - 54.5\,N$
$= 92.5\,N$

④ $F_f = \mu F_N$
$= (0.3)(92.5\,N)$
$= 27.75\,N$

⑤ $F_{net} = F_{a_x} - F_f$
$= 77.8\,N - 27.75\,N$
$= 50.05\,N$

⑥ $F_{net} = ma$
$a = \frac{F_{net}}{m}$
$= \frac{50.05\,N}{15\,kg}$
$\boxed{a = 3.3\,m/s^2}$

Case 3: Mass hanging off edge of table

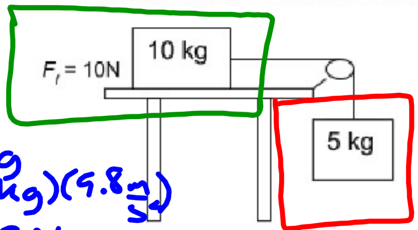
Remember: same a

- ~~Both masses~~ (or more) ~~accelerate together~~ because they are tied together.
- We ~~add the masses~~ to find the acceleration of the system.

$F_{net} = ma$

Examples: system — sum of masses

1. A 10 kg mass is tied to a 5 kg mass, as illustrated below. Friction exerts a force of 10 N. What is the acceleration of the 10 kg mass?



2 unknowns
→ a (same for both masses)
→ T (only one!)

$$F_{g(5)} = mg = (5\,kg)(9.8\,\tfrac{m}{s^2}) = 49\,N$$

Method #1: Solving using a system of equations (isolate each mass)

Isolate 10kg mass

F_N, F_f, T, F_g

$$F_{net} = T - F_f$$
$$ma = T - F_f$$
$$\boxed{(10\,kg)a = T - F_f}$$

system

Isolate 5kg mass

↓+ , T, F_g

$$F_{net} = F_{g(5)} - T$$
$$ma = F_{g(5)} - T$$
$$\boxed{(5\,kg)a = F_{g(5)} - T}$$

Solve the system!

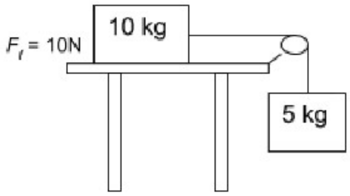
$$(10\,kg)a = T - F_f \qquad T = \boxed{F_{g(5)} - (5\,kg)a}$$
$$(10\,kg)a = F_{g(5)} - (5\,kg)a - F_f$$
$$(10\,kg)a + (5\,kg)a = F_{g5} - F_f$$
$$(\cancel{10\,kg + 5\,kg})\,a = F_{g5} - F_f \Rightarrow$$
$$a = \frac{49\,N - 10\,N}{(\cancel{10\,kg + 5\,kg})} = 2.6\,m/s^2$$

Method #2: Solving for the ~~system as a whole~~

For the system

$F_{net} = F_{accel} - F_{against\ motion}$

$F_{net} = (\text{total mass})\, a$



① $F_{g5} = 49N$

② $F_{net} = F_{accel} - F_{against}$
$= 49N - 10N$
$= 39N$

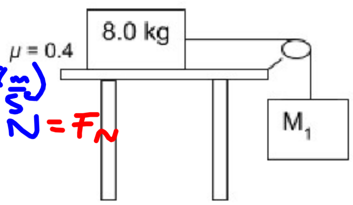
③ $F_{net} = ma$

$a = \dfrac{F_{net}}{m}$ ← entire system

$= \dfrac{39N}{\cancel{(10kg + 5kg)}}$

$= 2.6\ m/s^2$

2. The system below accelerates at 1.6 m/s^2. The coefficient of kinetic friction between the table and the 8.0 kg box is 0.4. What is the mass of M_1?



① $F_{g8} = mg$
$= (8kg)(9.8\frac{m}{s^2})$
$= 78.4N = F_N$

② $F_f = \mu F_N$
$= (0.4)(78.4N)$
$= 31.36N$

③ $F_{net} = ma$
$F_{net} = \boxed{(8.0kg + M_1)(1.6m/s^2)}$

④ $F_{net} = F_{accel} - F_{against}$
$F_{net} = F_{gM_1} - F_f$
$F_{net} = (M_1)(9.8\frac{m}{s^2}) - 31.36N$

$(8.0kg + M_1)(1.6\frac{m}{s^2}) = (M_1)(9.8\frac{m}{s^2}) - 31.36N$

$12.8N + (M_1)(1.6\frac{m}{s^2}) = (M_1)(9.8\frac{m}{s^2}) - 31.36N$

$44.16N = (M_1)(8.2\frac{m}{s^2})$

$M_1 = \dfrac{44.16N}{8.2m/s^2}$

$\boxed{M_1 = 5.4kg}$

Tension in between

To find the tension in the string that connects to objects together:

** • First we find the acceleration of the system.
• Then we isolate one of the masses, and apply Newton's second law.

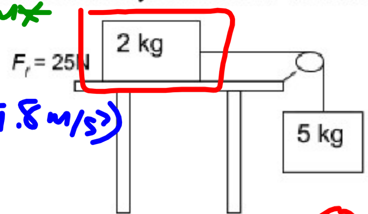
$F_{net} = ma$

one mass

Examples:

3. Consider the system below. What is the tension in the string?

Entire system



① $F_g = mg$
$= (5kg)(9.8 m/s^2)$
$= 49N$

② $F_{net} = F_{acc} - F_{against}$
$= F_{g5} - F_f$
$= 49N - 25N$
$= 24N$

③ $F_{net} = ma$
$a = \frac{F_{net}}{m}$
$= \frac{24N}{(2kg + 5kg)}$
$= 3.42 m/s^2$

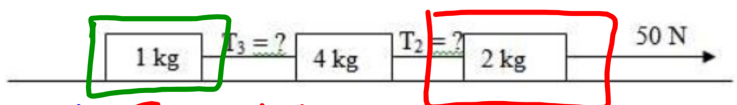
④ Isolate 2kg

a) $F_{net} = ma$
$= (2kg)(3.42 m/s^2)$
$= 6.84 N$

F_N, T, F_f, F_g

b) $F_{net} = T - F_f$
$T = F_{net} + F_f$
$= 6.84N + 25N = \boxed{31.84N = T}$

4. A series of masses are pulled along a frictionless surface, using a force of 50 N. What is the tension in strings 2 and 3?



① $F_{net} = 50N$
$F_{net} = ma$
$a = \frac{F_{net}}{m}$
$= \frac{50N}{(7kg)}$
$a = 7.14 m/s^2$

② Isolate 2kg
$F_{net} = ma$
$= (2kg)(7.14 m/s^2)$
$= 14.28N$

F_N, T_2, 50N, F_g

$F_{net} = 50N - T_2$
$T_2 = 50N - F_{net}$
$= 50N - 14.28N$
$T_2 = 35.72N$

③ Isolate 1kg
$F_{net} = ma$
$= (1kg)(7.14 \frac{m}{s^2})$
$= 7.14N$

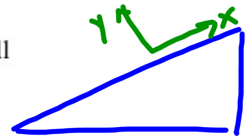
F_N, T_3, F_g

$F_{net} = T_3$
$T_3 = 7.14N$

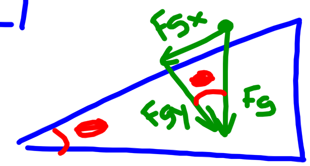
Case 4: Inclined Plane

When dealing with an inclined plane, we will call

- Direction parallel to motion: x
- Direction perpendicular to motion: y



Note: when splitting F_g, remember that it is always the hypotenuse.

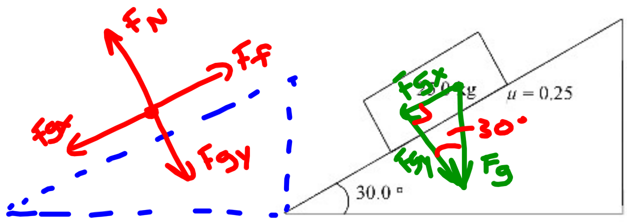


Examples:

*always redraw triangle

* $F_g \neq F_N$

1. What is the acceleration of the block down the incline?



① $F_g = mg$
$= (25\,kg)(9.8\,m/s^2)$
$= 245\,N$

$F_{g_x} = 245\,N \sin 30°$
$= 122.5\,N$

$F_{g_y} = 245\,N \cos 30°$
$= 212.2\,N$

② $F_N = F_{g_y}$ $(F_g \neq F_N)$
$F_N = 212.2\,N$

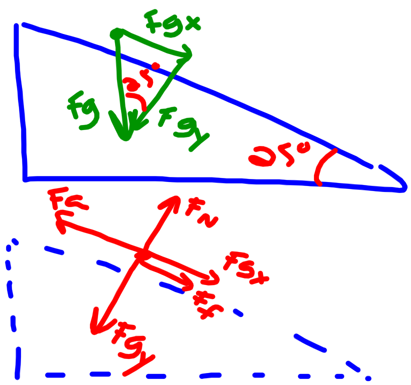
③ $F_f = \mu F_N$
$= (0.25)(212.2\,N)$
$= 53.05\,N$

④ $F_{net} = F_{g_x} - F_f$
$= 122.5\,N - 53.05\,N$
$= 69.45\,N$

⑤ $F_{net} = ma$
$a = \frac{F_{net}}{m}$
$= \frac{69.45\,N}{25\,kg}$
$= 2.8\,m/s^2$

2. A girl pushes a 50.0 kg box up a ramp set at 25 °. Friction exerts a force of 75 N. What is the magnitude of the force she must apply in order to slide the box at a constant velocity?

$\rightarrow F_{net} = 0$ (equilibrium)



① $F_g = mg$

$= (50\,kg)(9.8\,m/s^2)$

$= 490\,N$

$F_{gx} = 490\,N \sin 25°$

$= 207.1\,N$

$F_{gy} = 490\,N \cos 25°$

$= 444.1\,N$

② $F_{net} = 0$

$F_a = F_{gx} + F_f$

$= 207.1\,N + 75\,N$

$\boxed{F_a = 282.1\,N}$

b) What is μ?

$F_f = \mu F_N$

$\mu = \dfrac{F_f}{F_N}$ $\quad F_N = F_{gy}$

$= \dfrac{75\,N}{444.1\,N}$

$\mu = 0.17$

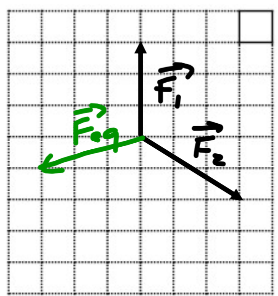
Equilibrant Force

The equilibrant force is the force that brings a system into equilibrium.

The equilibrant force has the same magnitude as the resultant (net) force, but it has opposite direction.

Examples:

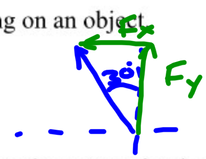
1. Two forces are simultaneously acting on an object, as illustrated below. Draw the equilibrant force.



$F_1\ (0,\ 3)$
$F_2\ (3,\ -2)$

$(3,\ 1)\ F_{net}$
$(-3,\ -1)\ F_{eq}$

2. The three forces below are acting on an object.
   $F_1 = 20.0$ N [S]
   $F_2 = 15.0$ N [E]
   $F_3 = 8.0$ N [N 30° W]



Equilibrium

What 4th force should be added to the system for the object to remain at rest?

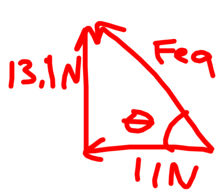
F_{eq}

$F_1\ (0,\ -20)$
$F_2\ (15,\ 0)$
$F_3\ (-4.0,\ 6.9)$

$(11,\ -13.1)\ F_{net}$
$(-11,\ 13.1)\ F_{eq}$

$F_x = 8.0\text{ N}\sin 30° = 4.0\text{ N}$

$F_y = 8.0\text{ N}\cos 30° = 6.9\text{ N}$



$\text{mag} = \sqrt{(11\text{N})^2 + (13.1\text{N})^2} = 17.1\text{N}$

$\theta = \tan^{-1}\left(\frac{13.1\text{N}}{11\text{N}}\right) = 50°$

$\boxed{\text{Ans: } 17.1\text{N [W50°N]}}$