

Chapter 6: Forces and Newton's Laws (Part I)

What is a force?

A force is a push or a pull. Forces affect motion. They can cause a body to change its shape, direction or velocity.

turning ← acceleration →

- Symbol: \vec{F}
Force is a vector, it has a magnitude and a direction.

- Units of Force: Newtons (N)

Note: $1\text{N} = 1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$

- To represent a force on a diagram: →

Types of forces

1) Gravitational Force

The force of attraction between any 2 bodies.

Symbol: F_g

mostly for star, planets, moons

Formula:

$$F_g = \frac{G \cdot m_1 \cdot m_2}{r^2}$$

G : gravitational constant ($6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$)

m_1 : one body's mass (in kg)

m_2 : other body's mass (in kg)

r : distance between the 2 bodies (in m)

from center of planet/moon/star

2) Simplified Gravitational Force (aka Weight)

Force due to gravity on bodies near the surface of the Earth

Symbol: F_g

Formula:

$$F_g = mg$$

m : mass (kg)

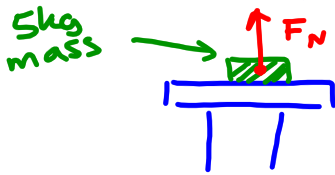
g : gravitational constant ($\frac{\text{N}}{\text{kg}}$, $\frac{\text{m}}{\text{s}^2}$)

on earth $g = 9.8 \frac{\text{m}}{\text{s}^2}$

3) Normal force → perpendicular

Force exerted by a surface, perpendicular to the surface.

Symbol: F_N (not one formula) = look at context



$$F_N = 49\text{ N} \quad \textcircled{1} \quad F_g = mg = (5\text{ kg}) \times (9.8 \frac{\text{m}}{\text{s}^2}) = 49\text{ N}$$

$$F_g = 49\text{ N}$$

$\textcircled{2} F_N = 49\text{ N}$

4) Force of friction

Force that opposes motion. It is a force between two surfaces that are in contact. (The direction is OPPOSITE to motion)

Symbol: F_f

Formula:

$$F_f = \mu F_N$$

$0 < \mu < 1$
 μ : coefficient of friction (no units)

↪ related to nature of surfaces

There are two types of friction:

- Static Friction (not moving)

↪ μ_s

- Kinetic Friction (sliding)

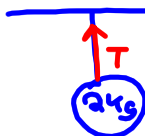
↪ μ_k

$$\mu_s > \mu_k$$

5) Tension

Force in a string/cable (as it is being pulled)

Symbol: T or F_T (no formula → context!)



$$T = 19.6\text{ N} \quad \textcircled{1} \quad F_g = mg = (2\text{ kg}) \times (9.8 \frac{\text{m}}{\text{s}^2}) = 19.6\text{ N}$$

$$F_g = 19.6\text{ N}$$

$\textcircled{2} T = 19.6\text{ N}$

6) Elastic force (Hooke's Law)

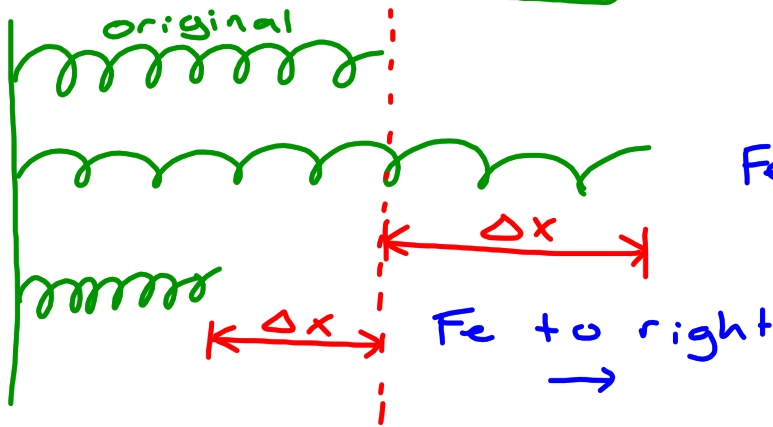
Force that tends to bring a spring back to its original length. (Remember that a spring can be stretched or compressed)

Symbol: F_e

Formula: $F_e = k \Delta x$

k : Spring constant (N/m)

Δx : deformation of spring (m)



7) Centripetal Force

Force that maintains a body in circular motion.

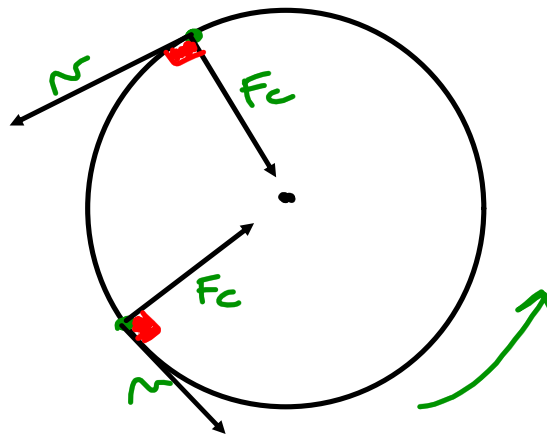
Symbol: F_c (F_c is always towards center of circle.)

Formula:

$$F_c = \frac{m v^2}{r}$$

m : mass (kg)
 v : linear (tangential) velocity (m/s)
 r : radius of circle (m)

velocity of object if you "let go" or no more F_c



8) Applied force

Force applied by a person, animal, engine, etc.

Symbol: F_a
or
 F_{eng} (no formula : - given OR - find it using context)

9) Net force (Resultant force)

Vector sum of all the forces acting on an object.
↔ direction matter

Symbol: F_{net} (Find F_{net} using - context OR - formula (coming soon!))

Free body diagrams

Free body diagrams (FBD) are used to show the forces acting on an object. You should always start problem solving by drawing a FBD.

In a FBD, objects are represented by: ●

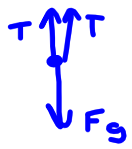
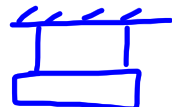
Arrows starting from this dot are used to show the forces acting on the objects.

Examples: Draw the FBD for the situations below

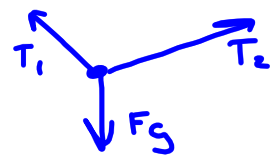
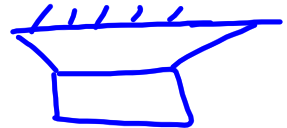
1) A box rests on a table.



2) A sign is supported by 2 vertical cables.



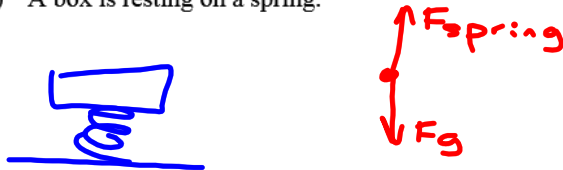
3) A sign is supported by 2 angled cables.



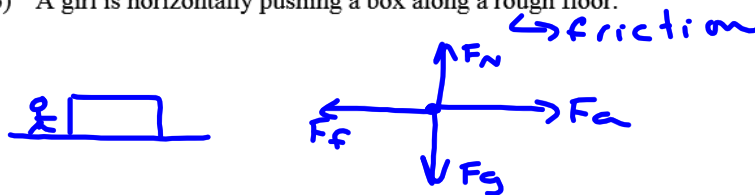
- 4) A box slides down a frictionless ramp.



- 5) A box is resting on a spring.



- 6) A girl is horizontally pushing a box along a rough floor.

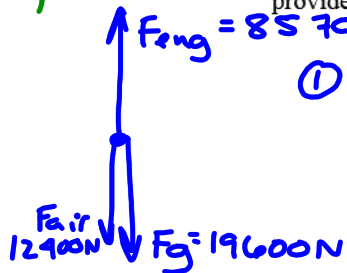


Finding the net force

To find F_{net} , we find the vector sum of all the forces exerted on one object.

Ex: The engine of a 2000. kg rocket provides 85 700 N of force. Air resistance provides 12 400 N. What is the net force on the rocket?

↑ +



$$\begin{aligned} \textcircled{1} F_g &= m g \\ &= (2000 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 19600 \text{ N} \end{aligned}$$

$$\begin{aligned} \textcircled{2} F_{net} &= F_{eng} - F_{air} - F_g \\ &= 85700 \text{ N} - 12400 \text{ N} - 19600 \text{ N} \\ &= 53700 \text{ N} \end{aligned}$$

(because +, up)

Equilibrium of forces

A system is said to be in equilibrium when $F_{net} = 0$.

When a system is in equilibrium,

- the object is not moving (at rest)
OR
- the object is moving at a constant velocity

When $F_{net} = 0$, it means that

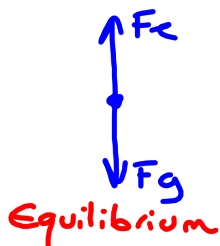
- all forces pushing the object "up" are equal to all forces pushing the object "down"
- all forces pushing the object "left" are equal to all forces pushing the object "right"

i.e. the vector sum of all forces is zero.

Knowing that a system is in equilibrium allows us to solve some problems.

Examples:

- 1) When you hang a 4.0 kg mass from a spring, the spring stretches by 10 cm. What is the spring constant of this spring?



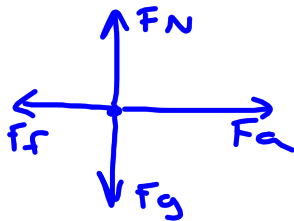
$$\textcircled{1} F_g = mg \\ = (4.0 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2}) \\ = 39.2 \text{ N}$$

$$\textcircled{2} F_e = 39.2 \text{ N (eq)}$$

$$\textcircled{3} F_e = k \Delta x \\ k = \frac{F_e}{\Delta x} \\ = \frac{39.2 \text{ N}}{0.1 \text{ m}}$$

$$k = 392 \text{ N/m}$$

- 2) A 45 kg box is being pushed along a rough floor. The coefficient of friction between the box and the floor is 0.20. What is the force of friction?



Vertical equilibrium

$$\textcircled{1} F_g = mg \\ = (45 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2}) \\ = 441 \text{ N}$$

$$\textcircled{2} F_N = 441 \text{ N}$$

$$\textcircled{3} F_f = \mu F_N \\ = (0.2)(441 \text{ N}) \\ = 88.2 \text{ N}$$

Newton's Laws

1st Law: Law of inertia

When no net force acts on an object, the object remains at rest or continues moving at a constant velocity.

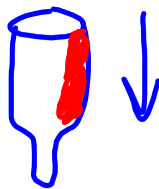
objects continue doing what they are already doing

Examples:

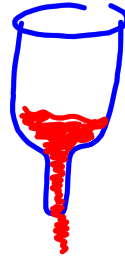
- 1) When you hit the brakes in a car, all objects that are not strapped down go flying forward. The objects continue moving at same velocity because no net force is acting on them.

↑ 1st law

- 2) Ketchup in a glass bottle!



down
force
and
stop!



bottle stops,
ketchup
keeps going

- 3) Heavier objects have more inertia

more mass = more inertia

2nd Law: Law of acceleration

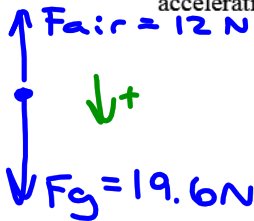
The acceleration of an object is directly proportional to the net force acting on the object, and inversely proportional to the mass of the object.

* (choose direction of F_{net} (a) as \oplus)

$$F_{net} = ma$$

* F_{net} and a are always in the same direction

- 1) A 2.0 kg rock falls from the edge of a cliff. Air resistance exerts 12 N. What is the acceleration of the rock?

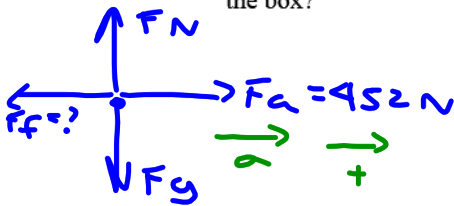


$$\begin{aligned} \textcircled{1} F_g &= mg \\ &= (2.0 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 19.6 \text{ N} \end{aligned}$$

$$\begin{aligned} \textcircled{2} F_{net} &= F_g - F_{air} \\ &= 19.6 \text{ N} - 12 \text{ N} \\ &= 7.6 \text{ N} \end{aligned}$$

$$\begin{aligned} \textcircled{3} F_{net} &= ma \\ a &= \frac{F_{net}}{m} \\ &= \frac{7.6 \text{ N}}{2.0 \text{ kg}} \\ a &= 3.8 \text{ m/s}^2 \end{aligned}$$

- 2) A 70.0 kg box is pushed along a rough surface with a horizontal force of 452 N. The box accelerates at a rate of 0.500 m/s^2 . What is the force of friction acting on the box?



$$\begin{aligned} \textcircled{1} F_{net} &= ma \\ &= (70.0 \text{ kg})(0.5 \text{ m/s}^2) \\ &= 35 \text{ N} \end{aligned}$$

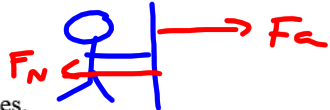
$$\begin{aligned} \textcircled{2} F_{net} &= F_a - F_f \\ F_f &= F_a - F_{net} \\ &= 452 \text{ N} - 35 \text{ N} \\ F_f &= 417 \text{ N} \end{aligned}$$

3rd Law: Action-Reaction

For every force applied to a object, the object applies a force of equal magnitude but opposite direction.

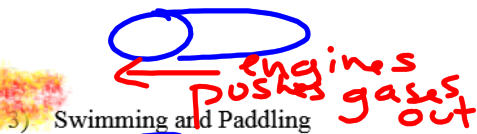
Examples:

- 1) You push the wall with 50 N, the wall pushed you back with 50 N



- 2) Jet engines.

propels plane



- 3) Swimming and Paddling



water pushes you forward

Solving problems using Newton's Second Law.

Basic Idea:

There are 2 ways of finding the net force

- $F_{net} = ma$ (given m and a)
- F_{net} = the vector sum of all forces
↳ using diagram

So to solve problems

- 1) Find F_{net} using both methods (you may get a "number" or an algebraic expression)
- 2) Make both F_{net} 's equal to each other
- 3) Solve for what you are looking for

Examples:

- 1) A boy pushed a 10 kg box along a rough surface. He applies a force of 125 N. The coefficient of friction between the floor and the box is 0.4. What is the acceleration of the box?

① $F_g = mg$
 $= (10 \text{ kg})(9.8 \text{ m/s}^2)$
 $= 98 \text{ N}$

② $F_N = F_g$
 $F_N = 98 \text{ N}$

③ $F_f = \mu F_N$
 $= (0.4)(98 \text{ N})$
 $= 39.2 \text{ N}$

④ $F_{net} = F_a - F_f$
 $= 125 \text{ N} - 39.2 \text{ N}$
 $= 85.8 \text{ N}$

⑤ $F_{net} = ma$
 $a = \frac{F_{net}}{m}$
 $= \frac{85.8 \text{ N}}{10 \text{ kg}}$
 $a = 8.58 \text{ m/s}^2$

- 2) A 2500 kg car's engine provides a force of 8500 N. The car accelerates at a rate of 2.6 m/s^2 . What is the magnitude of the force that slows down the car (air resistance combined with friction between parts)?

① $F_{net} = ma$
 $= (2500 \text{ kg})(2.6 \text{ m/s}^2)$
 $= 6500 \text{ N}$

② $F_{net} = F_{eng} - F_f$
 $F_f = F_{eng} - F_{net}$
 $= 8500 \text{ N} - 6500 \text{ N}$
 $= 2000 \text{ N}$

Some classics

Rocket Problems:

- Don't forget that gravity is involved
- Acceleration and velocity don't always have to be in the same direction

1) A 4.00 kg toy rocket is being propelled by its engine using a force of 50.0 N. What is the acceleration of the rocket?

$\uparrow a \uparrow +$

$F_{eng} = 50.0 \text{ N}$

$F_g = 39.2 \text{ N}$

① $F_g = mg$
 $= (4.00 \text{ kg})(9.8 \text{ m/s}^2)$
 $= 39.2 \text{ N}$

② $F_{net} = F_{eng} - F_g$
 $= 50.0 \text{ N} - 39.2 \text{ N}$
 $= 10.8 \text{ N}$

③ $F_{net} = m a$
 $a = \frac{F_{net}}{m} = \frac{10.8 \text{ N}}{4.00 \text{ kg}} = 2.7 \text{ m/s}^2$

2) The engine of a rocket applies a force of $3.0 \times 10^3 \text{ N}$. As a result, the rocket experiences an upward acceleration of 5.2 m/s^2 . What is the mass of the rocket?

$\uparrow a \uparrow +$

$F_{eng} = 3000 \text{ N}$

$F_g = mg$
 $F_g = m(9.8 \frac{\text{m}}{\text{s}^2})$

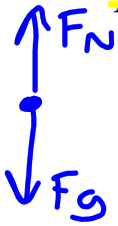
① $F_{net} = m a$
 $F_{net} = m (5.2 \text{ m/s}^2)$

② $F_{net} = F_{eng} - F_g$
 $m(5.2 \frac{\text{m}}{\text{s}^2}) = 3000 \text{ N} - m(9.8 \frac{\text{m}}{\text{s}^2})$
 $m(5.2 \frac{\text{m}}{\text{s}^2}) + m(9.8 \frac{\text{m}}{\text{s}^2}) = 3000 \text{ N}$
 $m(15 \frac{\text{m}}{\text{s}^2}) = 3000 \text{ N}$
 $m = \frac{3000 \text{ N}}{15 \frac{\text{m}}{\text{s}^2}}$
 $m = 200 \text{ kg}$

units

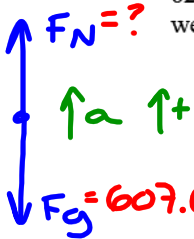
$\frac{\text{N}}{\frac{\text{m}}{\text{s}^2}} \rightarrow \frac{\text{kg} \frac{\text{m}}{\text{s}^2}}{\frac{\text{m}}{\text{s}^2}}$

Elevator Problems (standing on a scale in an elevator, of course)



- When the elevator has an upward acceleration
feels heavier ($F_N > F_g$)
- When the elevator has a downward acceleration
feels lighter ($F_N < F_g$)
- We use F_N as the force shown by the scale (assuming it is in Newtons)

3) As she often does, Carrie is standing on a scale in an elevator. Carrie has a mass of 62 kg. The elevator has an upward acceleration of 1.5 m/s^2 . What is Carrie's weight, according to the scale (which is in Newtons)?

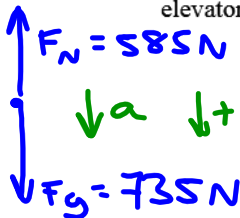


$$\begin{aligned} \textcircled{1} F_g &= mg \\ &= (62 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 607.6 \text{ N} \end{aligned}$$

$$\begin{aligned} \textcircled{2} F_{\text{net}} &= ma \\ &= (62 \text{ kg})(1.5 \text{ m/s}^2) \\ &= 93 \text{ N} \end{aligned}$$

$$\begin{aligned} \textcircled{3} F_{\text{net}} &= F_N - F_g \\ F_N &= F_{\text{net}} + F_g \\ &= 93 \text{ N} + 607.6 \text{ N} \\ \boxed{F_N} &= \boxed{700.6 \text{ N}} \end{aligned}$$

4) Tony, who has a mass of 75 kg, also likes to stand on scales in elevator. The elevator is moving, and he sees that scale indicated 585 N. What is the acceleration of the elevator? (Don't forget to specify the direction.)



$$\begin{aligned} \textcircled{1} F_g &= mg \\ &= (75 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 735 \text{ N} \end{aligned}$$

$$\begin{aligned} \textcircled{2} F_{\text{net}} &= F_g - F_N \\ &= 735 \text{ N} - 585 \text{ N} \\ &= 150 \text{ N} \end{aligned}$$

$$\begin{aligned} \textcircled{3} F_{\text{net}} &= ma \\ a &= \frac{F_{\text{net}}}{m} \\ &= \frac{150 \text{ N}}{75 \text{ kg}} \end{aligned}$$

$$\boxed{a = 2.0 \text{ m/s}^2 \text{ down}}$$

Using graph to determine a coefficient

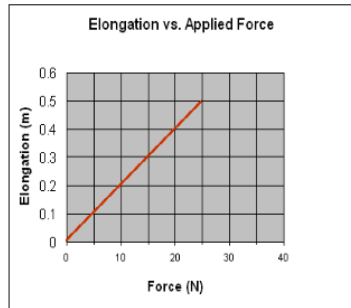
Sometimes we get graphs that allow us to determine either the coefficient of friction or the spring constant.

We find the rate of change (slope) or the inverse of the slope to determine the value of coefficient of friction or the spring constant.

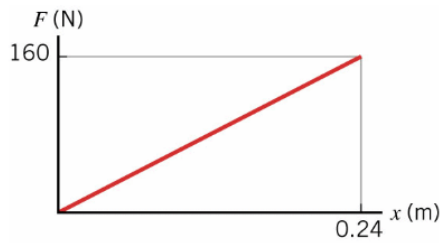
Examples:

**units don't lie*

1. Find the spring constant for each spring below.



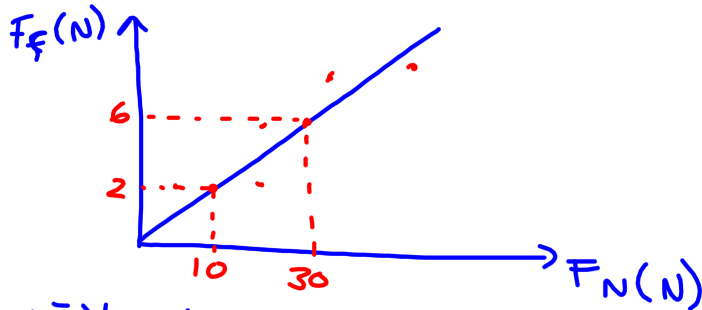
$$k = \frac{x_2 - x_1}{y_2 - y_1} \leftarrow N$$
$$= \frac{20N - 0}{0.4m} \leftarrow m$$
$$= 50N/m$$



$$k = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{160N - 0}{0.24m - 0}$$
$$= 667N/m$$

2. Find the coefficient for the surfaces represented below.

→ units are not helpful



$$\mu = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{6N - 2N}{30N - 10N}$$
$$= \frac{4N}{20N}$$

$$\mu = 0.2$$